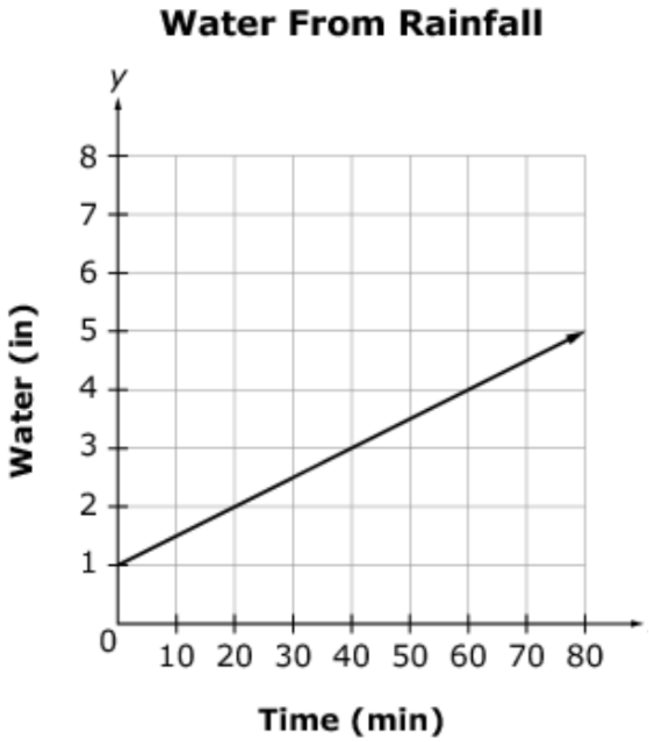
2

Jared measures the total height of the water in a bucket at different times during a rainfall. He displays his data using the graph shown.



Select **all** the conclusions that Jared can correctly make using the graph.

- ☐ The height of the water in the bucket increases $\frac{1}{2}$ inch every minute.
- ☐ The height of the water in the bucket increases at a rate of 3 inches per hour.
- ☐ In the first 20 minutes, the height of the water increases by 2 inches.
- ☐ It takes 60 minutes for the height of the water to increase by 4 inches.
- ☐ After the bucket has been outside for only 1 minute, there is no water in the bucket.
- ☐ There is already 1 inch of water in the bucket when Jared places it outside.

8.3.1 How can I divide polynomials?

Polynomial Division

When you graphed polynomial functions in the first section of this chapter, you learned that the factored form is useful for determining the zeros of the function or the x-intercepts of the graph. But when you do not have the factored form, what happens when you need to list all of the zeros or x-intercepts? You will investigate the answer to this question in this lesson.

8-84. Andre needs the exact x-intercepts of the function $f(x) = x^3 + 2x^2 - 7x - 2$. When he uses his graphing calculator, he sees that one of the x-intercepts is 2, but there are two other x-intercepts that he cannot identify exactly. What does he need to be able to do to determine the exact values of the other x-intercepts?

Andre remembers that he learned how to multiply binomials and other polynomials using an area model. He figures that since division is the inverse (or undo) operation for multiplication, he can reverse the area model multiplication process to divide polynomials. As he is thinking about his idea, he comes across the following news article.

Polydoku Craze Sweeping Nation!

(CPM) - Math enthusiasts around the nation have entered a new puzzle craze involving the multiplication of polynomials. The goal of the game, which enthusiasts have named Polydoku, is to fill in squares so that the multiplication of two polynomials will be completed.

	1	2	3	4	5
A	×	$2x^3$	$-x^2$	$+3x$	-1
B	$3x$	$6x^4$	$-3x^3$	$9x^2$	$-3x$
C	-2	$-4x^3$	$2x^2$	$-6x$	2

$6x^4$ $-7x^3$ $+11x^2$ $-9x$ $+2$

The game shown at right, for example, represents the multiplication of $(3x-2)(2x^3-x^2+3x-1)=6x^4-7x^3+11x^2-9x+2$.

Most of the squares are blank at the start of the game. While the beginner level provides the factors (in the gray squares), some of the factors are missing in the more advanced levels.

Analyze the game of Polydoku. How can Andre use the game to determine the other x-intercepts?

Team	8-85 (a)	8-85 (b)	8-85 (c)	8-86	8-87 Puzzle	8-87 Statement	8-88 (a)	8-88 (b)	8-88 (c)	8-89 (a)
1										
2										
3										
4										
5										
6										
7										
8										

Jiggy #30

Team	8-85 (a)	8-85 (b)	8-85 (c)	8-86	8-87 Puzzle	8-87 Statement	8-88 (a)	8-88 (b)	8-88 (c)	8-89 (a)
1										
2										
3										
4										
5										
6										
7										
8										

Day 1

Team	8-85 (a)	8-85 (b)	8-85 (c)	8-86	8-87 Puzzle	8-87 Statement	8-88 (a)	8-88 (b)	8-88 (c)	8-89 (a)
1			✗	✗						
2	✗	✗	✗	✗						
3										
4	✗	✗	✗	✗	✗					
5	✗	✗	✗	✗						
6										
7	✓	✓	✓	✓	✓		✓			
8										

8-85. Andre decides to join the craze and try some Polydoku puzzles, but he is not sure how to fill in some of the squares. Help him by answering parts (a) and (b) below about the Polydoku puzzle in the news article he read (found in problem 8-84); then complete part (c).

a. Explain how the term $2x^2$ in cell C3 of the news article was generated.

b. What values were combined to get $-7x^3$ in the news article answer?

c. Copy and complete the Polydoku puzzle below.

	1	2	3	4	5
A	$\times$	$4x^3$	$+ 6x^2$	$- 2x$	$- 5$
B	$2x$				
C	$- 3$				

8-86. POLYDOKU TEAM CHALLENGE

Work with your team to complete the puzzle at right. What are the factors and the product for the puzzle? If you get stuck, you can consult parts (a) through (c) below for ideas.

	1	2	3	4	5
A	×			$-2x$	
B	x	$2x^4$			
C	-4		$12x^2$		

$12x$

a. How is cell B2 related to the answer?

b. How did you determine the third term in the answer?

c. What cells did you use to get the value in cell B5?

8-87. Jessica is about to start the intermediate-level Polydoku puzzle shown at right. Show Jessica how to complete the puzzle. Make sure you can justify your solution.

Use your results to complete the statements below.

$$\frac{6x^3 + 7x^2 - 16x + 10}{2x + 5} = \_\_\_\_\_\_\_\_ \quad \text{and } (2x+5) \cdot \_\_\_\_\_\_\_\_ = \_\_\_\_\_\_\_\_$$

	1	2	3	4
A	×			
B	$2x$			
C	$+5$			

$6x^3$ $+7x^2$ $-16x$ $+10$

8-88. Unfortunately, Jessica made a mistake when she copied the puzzle for problem 8-87. The constant term of the original polynomial was supposed to have the value + 18 (not + 10). She does not want to start all over again to solve the puzzle.

a. Jessica realizes that she now has 8 remaining from the original expression. What is the significance of this 8 in relationship to the division problem?

$\frac{6+1}{3} = \frac{7}{3}$ $\quad 3\overline{)7}$ 2 R1 $= 2\frac{1}{3}$

b. Jessica writes her work as shown below:

$$\frac{6x^3+7x^2-16x+18}{2x+5} = \frac{(6x^3+7x^2-16x+10)+8}{2x+5} = 3x^2-4x+2 \text{, remainder } 8.$$

Gina thinks that there is a way to write the answer without using the word "remainder." Discuss this with your team and write the result another way. Be prepared to share your results and your reasoning with the class.

c. Use Jessica and Gina's method to divide $(6x^3 + 11x^2 - 12x - 1) \div (3x + 1)$.

$$\frac{6x^3+11x^2-12x-1}{3x+1}$$

$2x^2 \quad +3x \quad -5$

$3x$	$6x^3$	$9x^2$	$-15x$	R
$+1$	$2x^2$	$3x$	-5	4

$$= \frac{(6x^3+11x^2-12x-5)+4}{3x+1}$$

Sum: $6x^3+11x^2-12x-1$

$$= 2x^2+3x-5+\frac{4}{3x+1}$$

Day 2

Team	8-89 (a)	8-89 (b)	8-89 (c)	8-89 (d)	8-90	8-91 (a)	8-91 (b)	8-91 (c)	8-91 (d)	8-91 (e)
1										
2										
3										
4										
5										
6										
7										
8										

8-89. Create your own Polydoku puzzles that can be used to solve each of the polynomial-division problems below. Express any remainders as fractions and use your results to write a multiplication and a division statement like those in problem 8-88.

a. $\dfrac{6x^4-5x^3+10x^2-18x+5}{3x-1}$

b. $(x^4-6x^3+18x-4)\div(x-2)$

c. $x-3\overline{)x^3+x^2-14x+3}$

d. $\dfrac{x^5-1}{x-1}$

$x = 2$

8-90. Now work with your team to help Andre solve his original problem (problem 8-84). What are all of the x-intercepts of the graph of the polynomial?

$(x-2)$

8-84. Andre needs the exact x-intercepts of the function $f(x) = x^3 + 2x^2 - 7x - 2$. When he uses his graphing calculator, he sees that one of the x-intercepts is 2, but there are two other x-intercepts that he cannot identify exactly. What does he need to be able to do to determine the exact values of the other x-intercepts?

$(x-2)$

	x^2	$+4x$	$+1$	
x	x^3	$4x^2$	x	R
-2	$-2x^2$	$-8x$	-2	0

$$x^3 + 2x^2 - 7x - 2 = (x-2)(x^2+4x+1)$$

Roots → $0 = (x-2)(x^2+4x+1)$

$$x - 2 = 0 \qquad x^2 + 4x + 1 = 0$$

$$x = 2 \qquad x^2 + 4x + 4 = 3 \quad (+3 \;\; +3)$$

$$(x+2)^2 = 3$$

$$|x+2| = \sqrt{3}$$

$$x = -2 \pm \sqrt{3}$$

8-91. Polynomial division $\frac{P(x)}{D(x)}$ (where $D(x) \neq 0$) can be used to change any polynomial in standard form into the form $P(x) = D(x) \cdot Q(x) + R$. Use polynomial division to rewrite each polynomial in this form given the divisor, $D(x)$.

a. $P_1(x) = 2x^3 - 21x^2 - 9x - 30$
$D_1(x) = x - 11$

b. $P_2(x) = 3x^4 + 5x^3 - 4x + 3$
$D_2(x) = x + 2$

c. Use the original polynomial in part (a) to evaluate $P_1(11)$. Then use your rewritten polynomial to evaluate $P_1(11)$. What do you notice?

d. Use the original polynomial in part (b) to evaluate $P_2(-2)$. Then use your rewritten polynomial to evaluate $P_2(-2)$. What do you notice?

e. The **Remainder Theorem** states that if a polynomial $P(x)$ is divided by $(x - c)$, then the remainder is the value of $P(c)$. Why do you think this is true?

f. What can you conclude if $P(x)$ is divided by $(x - c)$ and the remainder is 0?