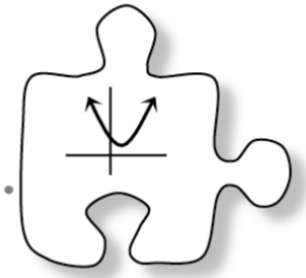
8.2.2 What do I know about the roots?

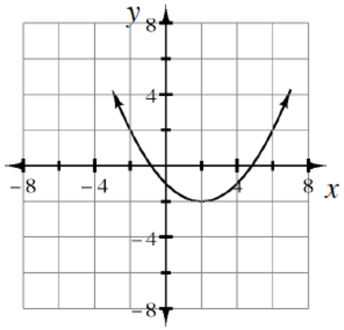


More Real and Complex Roots

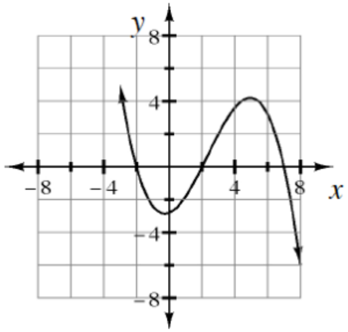
In the previous lesson, you reviewed the relationships between the graphs of quadratic functions and their roots, and then you used the roots of quadratic functions to write their equations. In this lesson, you will continue to explore the connections among the roots, the equations, and the graphs of polynomial functions.

8-72. Based on the following graphs, how many *real roots* does each polynomial function have? Justify your reasoning.

a. 2



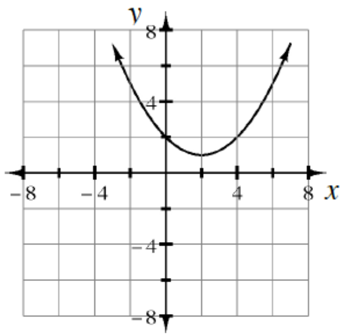
b. 3



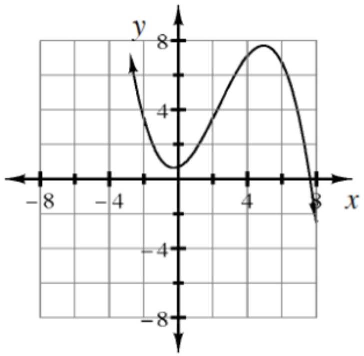
Do the polynomials in parts (c) and (d) have fewer roots than the corresponding polynomials in parts (a) and (b)?

Graphs (a) and (b) above have been vertically shifted to create graphs (c) and (d) below. How many *real roots* does each of these new polynomial functions have?

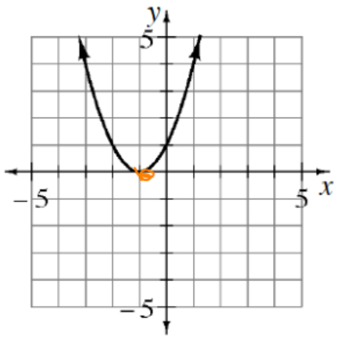
c. 0



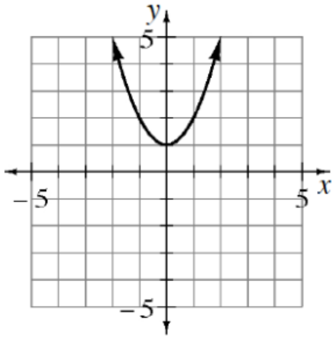
d. 1



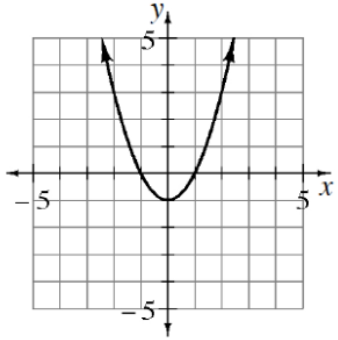
8-73. Recall that the graph of a polynomial function with degree n intersects the x-axis at most n times. For example, consider the three graphs below. The graph of $y = (x + 1)^2$ intersects the x-axis once, while $y = x^2 + 1$ does not intersect the x-axis. The function $y = x^2 - 1$ intersects the x-axis twice.



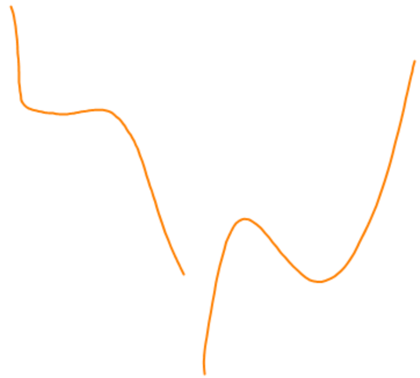
$y = (x + 1)^2$

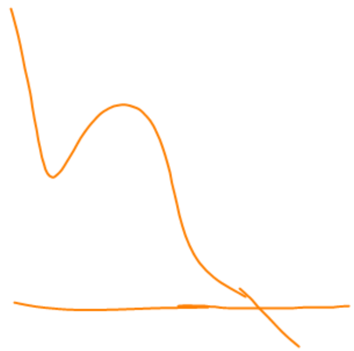


$y = x^2 + 1$



$y = x^2 - 1$





a. How many times can a polynomial of degree 3 intersect the x-axis? Make a sketch of each possibility.

b. Can a third-degree polynomial have no real roots? Explain why or why not.

8-74. Now consider the polynomial $y = x^3 - 3x^2 + 3x - 2$.

a. How many real roots could the polynomial $y = x^3 - 3x^2 + 3x - 2$ have?

b. Verify that $x^3 - 3x^2 + 3x - 2 = (x - 2)(x^2 - x + 1)$. Can it be factored further? No

Yes ✓

c. Solve for all the roots of $y = x^3 - 3x^2 + 3x - 2$. $2,\ \frac{1}{2} + \frac{i\sqrt{3}}{2},\ \frac{1}{2} - \frac{i\sqrt{3}}{2}$

d. How many x-intercepts does the graph of $y = x^3 - 3x^2 + 3x - 2$ have? How many real roots and how many complex roots does the polynomial have? $y = x^3 - 3x^2 + 3x - 2$

$0 = (x-2)(x^2 - x + 1)$

$x - 2 = 0$

$x = 2$

$x^2 - x + 1 = 0$

$x = \frac{+1 \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$

$x = \frac{1 \pm \sqrt{-3}}{2}$

$x = \frac{1 \pm i\sqrt{3}}{2}$

$x = \frac{1}{2} \pm \frac{i\sqrt{3}}{2}$

8-75. For each polynomial function below, determine all of the roots and describe them completely. Then sketch a graph of the function.

a. $P_1(x) = x(x^2 + x - 1)$

$0 = x(x^2+x-1)$

$x=0$ $\quad x^2+x-1=0$

$x = \frac{-1 \pm \sqrt{1^2-4(1)(-1)}}{2(1)}$

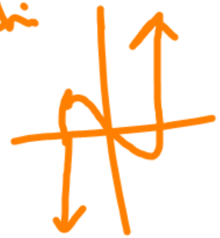
$x = \frac{1 \pm \sqrt{5}}{2}$

Roots: $x=0$

$x = 1.61$

$x = -1.61$

Sketch:



b. $P_2(x) = (x + 1)(5x^2 + x + 1)$

c. $P_3(x) = (x^2 + 1)(x^2 - 4)$

$0 = (x^2+1)(x^2-4)$

$0 = x^2+1$ $\quad 0 = x^2-4$

$-1 \quad -1$

$-1 = x^2$

$\sqrt{-1} = |x|$

$0 = (x-2)(x+2)$

$x=2 \quad x=-2$

$x = \pm i$

d. $P_4(x) = (x^2 + x - 2)(x^2 + x - 1)$

e. Describe any relationships you noticed between the equations of the polynomials and their roots.

b) $0 = (x+1)(5x^2+x+1)$

$x+1=0$ $\quad 5x^2+x+1=0$

$x=-1$

$x = \frac{-1 \pm \sqrt{(1)^2-4(5)(1)}}{2(5)}$

$x = \frac{-1 \pm \sqrt{1-20}}{10}$

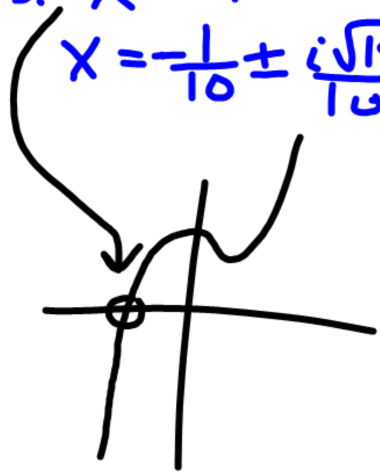
$x = \frac{-1 \pm \sqrt{-19}}{10}$

$x = \frac{-1 \pm i\sqrt{19}}{10}$

$x = \frac{-1}{10} \pm \frac{i\sqrt{19}}{10}$

Roots: $x = -1$

$x = -\frac{1}{10} \pm \frac{i\sqrt{19}}{10}$



	1	2	3	4	5
A	$\times$	$2x^3$	$-x^2$	$+3x$	-1
B	$3x$	$6x^4$	$-3x^3$	$9x^2$	$-3x$
C	-2	$-4x^3$	$2x^2$	$-6x$	2

$\mathbf{6x^4 \quad -7x^3 \quad +11x^2 \quad -9x \quad +2}$

	1	2	3	4	5
A	$\times$			$-2x$	
B	x	$2x^4$			
C	-4		$12x^2$		

$\mathbf{12x}$