

8.1.1 How can I describe the graph?

Sketching Graphs of Polynomial Functions



In previous courses and chapters, you learned how to graph many types of functions, including lines and parabolas. Today, you will work with your team to apply your graphing knowledge to graph more complicated polynomial functions. Just as quadratic polynomial functions can be written in standard or factored form, other polynomial functions can be written in standard or factored form. For example, $y = x^4 - 4x^3 - 3x^2 + 10x + 8$ is in standard form, but it can be written in factored form as $y = (x + 1)^2(x - 2)(x - 4)$.

During this lesson, you will develop techniques for sketching the graph of a polynomial function from its equation, and you will justify why these techniques work.



8-1. The Mathamericaland Carnival Company has decided to build a new roller coaster to use at this year's county fair. The new coaster will have a very special feature: part of the ride will be underground. The designers will use polynomial functions to describe different pieces of the track. Part of the design is shown at right. Your task is to guess a possible equation to represent the track and test it on your graphing calculator. To help get an idea of what to try, start by checking the graphs of the equations given below. Think about how the graphs are the same and how they are different.

$$y = x(x - 2)$$

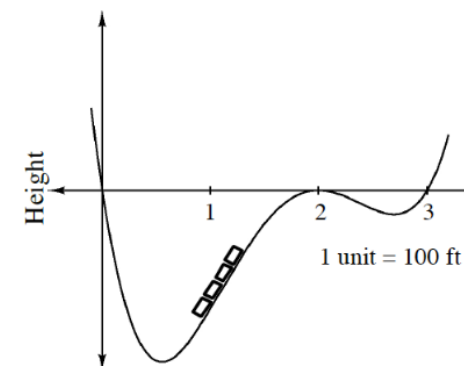
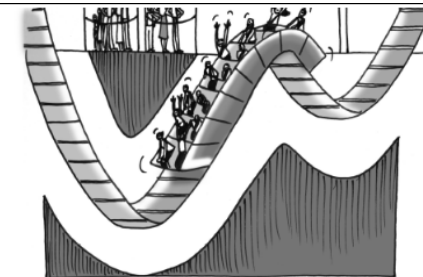
$$y = (x - 2)^2$$

$$y = x(x - 2)(x - 3)$$

Your Task: Use the information you gathered by graphing the above equations to help you make guesses about the equation that would produce the graph of the roller coaster. Once you have a graph that has a shape close to this one, try zooming in or changing the viewing window on your graphing calculator to see the details better. Keep track of what you tried and the equations that fit most accurately.

8-2. POLYNOMIAL FUNCTION INVESTIGATION

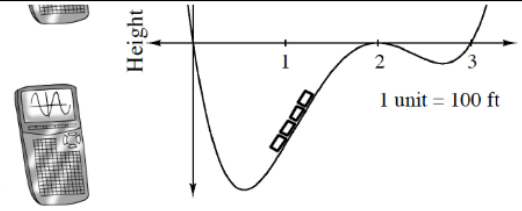
In this investigation, you will determine which information in the equation of a polynomial function can help you sketch its graph.



8-2. POLYNOMIAL FUNCTION INVESTIGATION

In this investigation, you will determine which information in the equation of a polynomial function can help you sketch its graph.

Your Task: With your team, write a thorough description, including examples and justifications, of the relationship between a polynomial function and its graph. To accomplish this task, first divide up the functions listed below so that each team member is responsible for two or three of them. Make a complete graph of each of your functions. Whenever possible, start by making a sketch without using your graphing calculator. Then, as a team, share your observations including your responses to the Discussion Points that follow. Then, choose two or three functions that can be used to represent all of your results. You can choose them from the list below, or you can create new ones as a team.



Create a presentation for your class. Be sure you include complete graphs and descriptions.

$$P_1(x) = (x - 2)(x + 5)^2$$

$$P_2(x) = 2(x - 2)(x + 2)(x - 3)$$

$$P_3(x) = x^4 - 21x^2 + 20x$$

$$P_4(x) = (x + 3)^2(x + 1)(x - 1)(x - 5)$$

$$P_5(x) = -0.1x(x + 4)^3$$

$$P_6(x) = x^4 - 9x^2$$

$$P_7(x) = 0.2x(x + 1)(x - 3)(x + 4)$$

$$P_8(x) = x^4 - 4x^3 - 3x^2 + 10x + 8$$

Discussion Points

What can we predict from looking at the equation of a polynomial function?
Why does this make sense?

Which form of a polynomial function is most useful for making a graph?
What information does it give?

How can we use the equation to help predict what a useful graphing calculator window might be?

Which examples are most helpful in making the connections between the equation and the graph?

8-3. As a team, examine the first polynomial $P_1(x) = (x - 2)(x + 5)^2$.

- To which family of functions does it belong? How do you know? Based on its equation, sketch the shape of its graph.
- Now use your graphing calculator to graph $y = P_1(x)$. Label the x -intercepts. How are the x -intercepts related to the equation? “Reading” from left to right along the x -axis, describe the graph.



8-4. Continuing your work as a team, examine the polynomial $P_2(x) = 2(x - 2)(x + 2)(x - 3)$.

- How many distinct (different) linear factors are there? How many x -intercepts do you predict it will have on its graph? Sketch the graph and label the x -intercepts. How is this graph similar to or different from the graph of $y = P_1(x)$?
- Does the constant factor 2 have any effect on the x -intercepts? On the shape of the graph? On the y -intercepts? How does the graph change if the factor 2 is changed to -2 ?

8-5. What is different about $P_3(x) = x^4 - 21x^2 + 20x$? What x -intercept(s) can you determine from the equation before graphing with the calculator? Explain how you know. Use the graph to figure out exactly what the other intercepts are. Explain how you can show that your answers are exact.

8-6. With your team, divide up the work to investigate $P_4(x)$ through $P_8(x)$ and continue your investigation, referring back to the “Your Task” statement and the Discussion Points in problem 8-2.