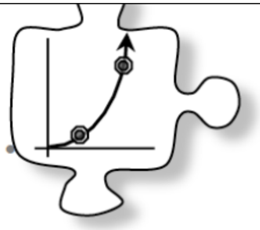
7.1.3 How can I write an exponential function?



Writing Equations of Exponential Functions

You have worked with logarithms throughout this chapter. Today you will look at how you can use logarithms to analyze situations that can be modeled using exponential functions.

Team 1	Team 2	Team 3	Team 4	Team 5	Team 6	Team 7	Team 8

7-31. DUE DATE

Brad's mother has just learned that she is pregnant! Brad is very excited that he will soon become a big brother. However, he wants to know when his new sibling will arrive and decides to do some research. On the Internet, he finds the following article:

Hormone Levels for Pregnant Women

When a woman becomes pregnant, the hormone HCG (human chorionic gonadotropin) is produced to enable the baby to develop.

During the first few weeks of pregnancy, the level of HCG hormone grows exponentially, starting with the day the embryo is implanted in the womb. However, the rate of growth varies with each pregnancy. Therefore, doctors cannot use just a single test to determine how long a woman has been pregnant. They must test the levels over time. Commonly, the HCG levels are measured two days apart to look for this rate of growth.

Brad's mother says she was tested for HCG during her last two doctor visits. On March 21, her HCG level was 200 mIU/ml (milli-international units per milliliter). Two days later, her HCG level was 392 mIU/ml.

a. Assuming that the model for HCG levels is of the form $y = ab^x$, write an equation that models the growth of HCG for Brad's mother's pregnancy.

b. Most women maintain an HCG level of 5 mIU/ml before becoming pregnant. Assuming that Brad's mother's level of HCG on the day of implantation was 5 mIU/ml, on what day did the embryo most likely become implanted? How many days after implantation was his mother's first doctor visit?

c. Brad learned that a baby is born approximately 38 weeks after implantation. When can Brad expect his new sibling to be born?

$y = ab^x$

a - intial value
b - multiplier
x - Day
y - HCG Levels

	x	y
Mar 21	1	200
	2	
Mar 23	3	392

$200 = ab^1 \rightarrow \frac{200}{b^1} = a$

$392 = ab^3$

$392 = \left(\frac{200}{b}\right)b^3$

$392 = 200b^2$

$\sqrt{1.96} = \sqrt{b^2}$

$1.4 = b$

$\frac{200}{1.4} = a$

$142.857 = a$

$y = 142.857(1.4)^x$

x	$200 \cdot 1.4^x$
−11	4.9388019
−10	6.9143226
−9	9.6800516
−8	13.552072
−7	18.972901
−6	26.562062
−5	37.186886
−4	52.061641
−3	72.886297
−2	102.04082
−1	142.85714
0	200
1	280
2	392
3	548.8

×1.4
×1.4

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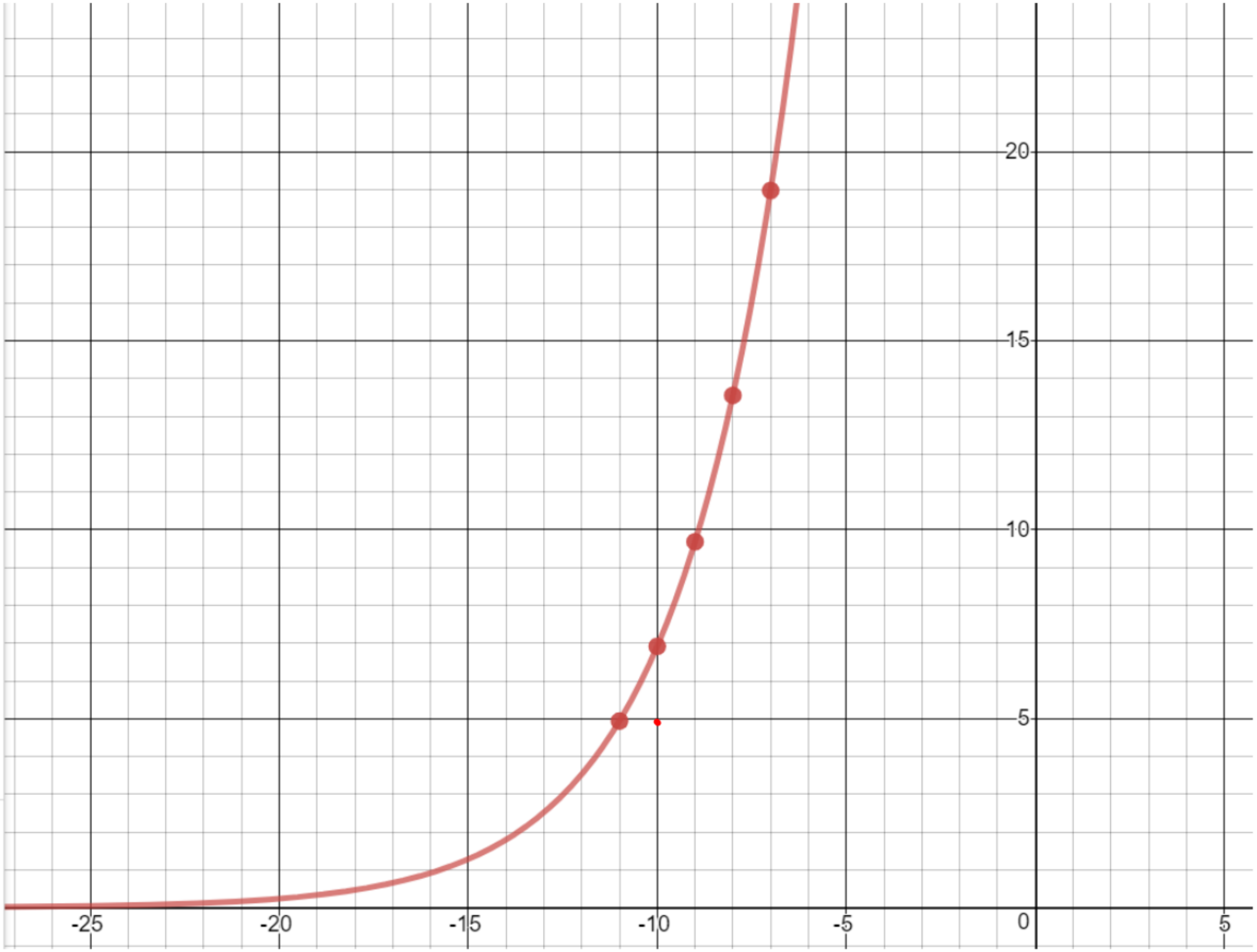


7-32. SOLVING STRATEGIES

In problem 7-31, you and your team developed a strategy to write the equation of an exponential equation of the form $y = ab^x$ when given two points on the curve.

a. What different strategies were generated by the other teams in your class? If no one shares your solving method with the class, be sure to share yours. Take notes on the different strategies students present.

b. Did any team use a system of exponential equations to solve for a and b? If not, examine this strategy as you answer the questions below.

i. The doctor visits provide two data points that can help you write an exponential model: (21, 200) and (23, 392). Use each of these points to substitute for x and y into $y = ab^x$. You should end up with two equations in terms of a and b.

ii. Consider the strategies you already have for solving systems of equations. Are any of those strategies useful for this problem? Discuss a way to solve your system from part (*i*) for a and b with your team. Be ready to share your method with the class.

i. The doctor visits provide two data points that can help you write an exponential model: (21, 200) and (23, 392). Use each of these points to substitute for x and y into $y = ab^x$. You should end up with two equations in terms of a and b.

7-33. The situation in problem 7-31 required you to assume that the exponential model had an asymptote at $y = 0$ to write the equation of the model. But what if the asymptote is not at the x-axis? Consider the situation below.

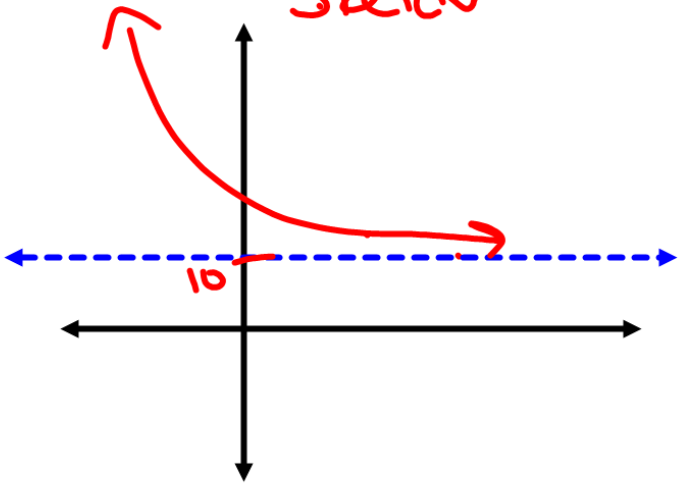
a. Assume the graph of an exponential function passes through the points (3, 12.5) and (4, 11.25). Is the exponential function increasing or decreasing? Justify your answer.

b. If the horizontal asymptote for this function is the line $y = 10$, make a sketch of its graph showing the horizontal asymptote.

c. If this function has the equation $y = ab^x + k$, what is the value of k? Use what you know about this function to write its equation. Verify that as x increases, the values of y get closer to $y = 10$.

d. What is the y-intercept of the function? What is the connection between the y-intercept and the asymptote?

Sketch



$y = ab^x + 10$

$12.5 = ab^3 + 10$

$11.25 = ab^4 + 10$

$12.5 = \left(\frac{1.25}{b^4}\right)b^3 + 10$

$12.5 = \frac{1.25}{b} + 10$

$-10 \qquad -10$

$b(2.5) = \left(\frac{1.25}{b}\right)b$

$2.5b = 1.25$

$b = .5$

$11.25 = ab^4 + 10$

$-10 \qquad -10$

$\frac{1.25}{b^4} = \frac{ab^4}{b^4}$

$\frac{1.25}{b^4} = a$

$a = 20$

$y = 20(.5)^x + 10$