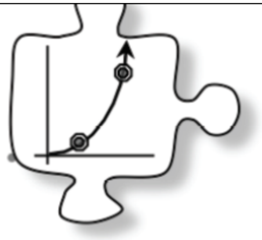
7.1.2 How can I rewrite it?



Investigating the Properties of Logarithms

You have explored the basic laws of exponents. Since logs are the inverses of exponential functions, they also have properties that are similar to the exponent properties you already know. In this lesson, you will explore these properties for logarithms.

7-15. Marta uses an old calculator that can only calculate logarithms with base 10. Marta now knows that if she wants to calculate $\log_2(30)$, she cannot type $\log(2^{30})$ into her calculator, because those expressions are not equivalent. But she still wants to know the value of $\log_2(30)$.

a. First, use your knowledge of logs and exponents to estimate $\log_2(30)$.

b. Now use what you learned in Lesson 7.1.1 to get a better estimate. Since you want to determine what $\log_2(30)$ equals, you can start by writing $\log_2(30) = x$. How can you use the definition of logarithm to rewrite this equation?

c. Use the methods you have developed to solve this equation. Refer back to your work from problem 7-6 if you need help.

$\frac{\log(30)}{\log(2)} = 4.906890596$

$2^{4.90689...} = 30$

$\log_2(30)$

$= 4.90689059561$

$\log_2(30) = x$

$2^x = 30$

$\log(2^x) = \log(30)$

Rewriting with Power Property

$x \cdot \log(2) = \log(30)$

$x = \frac{\log(30)}{\log(2)}$

$x \approx 4.907$

$\log_2(30) = \frac{\log(30)}{\log(2)}$

$\log_5(200) = x$

7-16. Congratulations! You have just used the log base 10 function to evaluate a log with base 2! Now you will practice some more.

a. First estimate an answer; then apply the method you have just developed to evaluate $\log_5(200)$.

b. Apply the process you used in part (a) to rewrite the expression $\log_b(y) = x$ in exponential form and then solve for x. This is called the **change of base** formula.

$\log_5(200) = x$

$5^x = 200$

$\log(5^x) = \log(200)$

$x \cdot \log(5) = \log(200)$

$x = \frac{\log(200)}{\log(5)}$

Change of Base:

$\log_b(y) = \frac{\log(y)}{\log(b)}$

$b^x = y$

$\log(b^x) = \log(y)$

$x \cdot \log(b) = \log(y)$

$x = \frac{\log(y)}{\log(b)}$

Team	7-17 (a)	7-17 (b)	7-17 (c)	Team Build	Log Puzzles RP	7-19	7-20 (a)	7-20 (b)	7-20 (c-d)	7-20 (e-f)
1										
2										
3										
4										
5										
6										
7										

Product Property of Logarithms

$$\log(m) + \log(n) = \log(m \cdot n)$$

Quotient Property of Logarithms

$$\log(m) - \log(n) = \log\left(\frac{m}{n}\right)$$

7-17. Since the log function is the inverse of an exponential function, the properties of exponents also apply to logarithms. The problems below will help you discover how exponent properties apply to logarithms.

a. Complete the two exponent rules below.

$x^m x^n =$ x^{m+n} and $\frac{x^m}{x^n} =$ x^{m-n}

b. To help you write the equivalent log properties, use your calculator to solve for x in each problem below. Note that x is a whole number in parts (i) through (v). Look for patterns that will make your job easier and allow you to generalize in part (vi).

i. $\log(5) + \log(6) = \cancel{\log(x)}$ $\log(5\cdot 6)$

ii. $\log(5) + \log(2) = \log(x)$

iii. $\log(5) + \log(5) = \cancel{\log(x)}$ $\log(25)$

iv. $\log(10) + \log(100) = \log(x)$

v. $\log(9) + \log(11) = \cancel{\log(x)}$ $\log(49)$

vi. $\log(m) + \log(n) = \log(m\cdot n)$

c. What if the log expressions are being subtracted instead of added? Solve for x in each problem below. Note that x will not always be a whole number. Again, look for patterns that will allow you to generalize in part (vi).

i. $\log(20) - \log(5) = \log(4)$

ii. $\log(30) - \log(3) = \log(10)$

iii. $\log(5) - \log(2) = \log(\frac{5}{2})$

iv. $\log(17) - \log(9) = \log(x)$

v. $\log(375) - \log(17) = \log(x)$

vi. $\log(m) - \log(n) = \log(\frac{m}{n})$

What patterns did you see when two log expressions were added?

How could you generalize the process?

What about for subtracting two log expressions?

What relationships are there between these new log properties and properties of exponents?