7.1.1 How can I solve exponential equations?

Using Logarithms to Solve Exponential Equations

In Chapter 5, you learned what a logarithm is and several important facts about logs. In this lesson, you will learn about a property of logarithms that will be very useful for solving problems that involve variable exponents.

7-1. LOGARITHMS SO FAR

You have learned three important log facts so far. Review these facts by discussing the questions below with your team to ensure everyone remembers these ideas. For each part, make up an example to illustrate your ideas.

a. What is a logarithm? How can log equations be converted into another form?

b. What do you know about the logarithm key on your calculator?

c. What does the graph of $y = \log(x)$ look like? Write the graphing form equation for $y = \log(x)$.

Answers:

Logarithms are a quantity representing the power to which a fixed number must be raised to produce a given number.

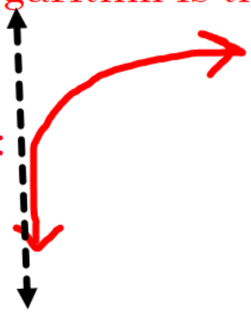
The logarithm is the inverse of an exponential function.

$y = \log(x)$

(h, k)

$(0, 1)$

The graph:



The base of log in the calculator is 10.

The exponential equivalent of y = log(x) is 10^y = x

$$y = a \cdot \log_b(x-h) + k$$

2. Marta wants to graph $y = \log_2(x)$ on her graphing calculator. She types in $y = \log(2^x)$ and presses $\boxed{\text{GRAPH}}$.

$2^y = x$

"Wow, that's not the graph I expected." Marta says.

$y = 2^x$ $\quad$ $y = \log_2(x)$

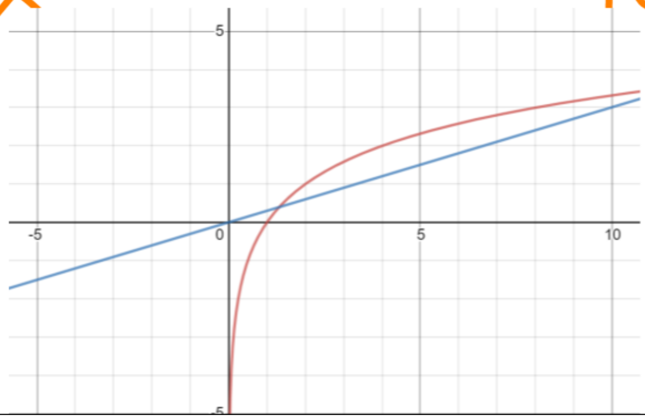
"Hmmm …," says Celeste. *"I think* $y = \log_2(x)$ *and* $y = \log(2^x)$ *are totally different. How can we prove that?"*

a. Show that the two equations are different by sketching the graph of $y = \log_2(x)$. Then sketch what your graphing calculator shows to be the graph of $y = \log(2^x)$.

b. Now show that $y = \log_2(x)$ and $y = \log(2^x)$ are different by converting both of them to exponential form.

$y = \log_2(x)$ $\qquad$ $y = \log(2^x)$

$2^y = x$ $\qquad$ $10^y = 2^x$

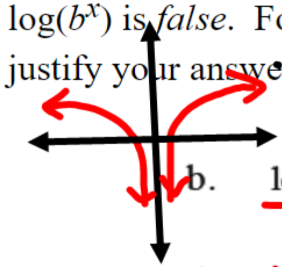


7-3. The work you did in problem 7-2 is a **counterexample**. By demonstrating that $\log_2(x)$ is not equivalent to $\log(2^x)$, you have shown that in general, the statement $\log_b(x) = \log(b^x)$ is *false*. For each of the following log statements, use the strategies from problem 7-2 to determine whether they are true or false, and justify your answer. Be ready to present your conclusions and justifications.

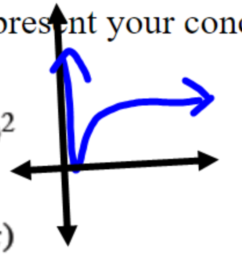
$\log_b(x) = y$

$b^y = x$

a. $\log_5(25) \overset{?}{\neq} \log_{25}(5)$



b. $\log(x^2) \overset{?}{\neq} (\log x)^2$



c. $\log(7^x) \overset{?}{=} x\log(7)$ ✓

d. $\log(2x) \overset{?}{=} \log_2(x)$

a.

$\log_5(25) = x$

$5^x = 25$

$\log_{25}(5) = x$

$25^x = 5$

b.

$\log(x^2) = y$

$10^y = x^2$

$(\log(x))^2 = y$

$\log(x) = \sqrt{y}$

$10^{\sqrt{y}} = x$

$\neq$

7-4. In the previous problem only *one* of the statements is true.

a. Use different numbers to make up four more statements that follow the same pattern as the one true statement, and test each one to see whether it appears to be true.

b. Use your results to complete the following statement, which is known as the **Power Property of Logarithms**: $\log(m^n) =$ ________ .

$$\log(7^x) = x \cdot \log(7)$$

Power Property of Logarithms: $\log(m^n) = \underline{n \cdot \log(m)}$.

$1.04^x = 2$

$x = 17.\sim$

$\log(1.04^x) = \log(2)$

Rewrite

$\frac{x \cdot \log(1.04)}{\log(1.04)} = \frac{\log(2)}{\log(1.04)}$

$10^2 = 100$

$\log(10^2) = \log(100)$

$2 \cdot \log(10) = 2$

$2 = 2$ ✓

$x = \frac{\log(2)}{\log(1.04)}$

$x \approx 17.673$

7-6. THERE MUST BE AN EASIER WAY

It would certainly be helpful to have an efficient method to solve equations like $1.04^x = 2$. Complete parts (a) through (c) below to discover an easier way.

$2^x = 9$

a. What makes the equation $1.04^x = 2$ so hard to solve?

b. Surprise! In the first part of this lesson, you already found a method for rewriting equations with inconvenient exponents! Talk with your team about how your results from problems 7-3 and 7-4 can help you rewrite the equation $1.04^x = 2$. Be prepared to share your ideas with the class.

c. Solve $1.04^x = 2$ using this new method. Be sure to check your answer.

$$1.04^x = 2$$

$$\log(1.04^x) = \log(2)$$

Power Prop. of Logs

$$\frac{x \cdot \log(1.04)}{\log(1.04)} = \frac{\log(2)}{\log(1.04)}$$

Exact $\rightarrow$ $x = \dfrac{\log(2)}{\log(1.04)}$

Approx $\rightarrow$ $x \approx 17.673$

7-7. Solve the following equations. After checking your answers, round them to three decimal places.

a. $5 = 2.25^x$

b. $3.5^x = 10$

c. $2(8^x) = 128$

d. $2x^8 = 128$