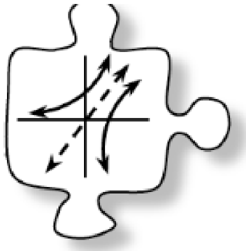
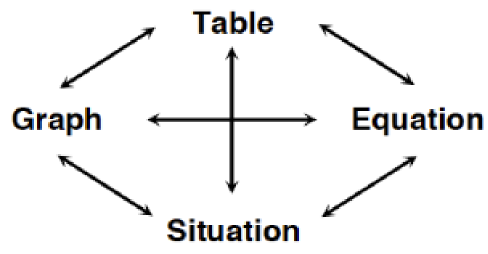
5.2.1 How can I undo an exponential function?



The Inverse of an Exponential Function

When you first began investigating exponential functions, you looked at how their different representations were connected, as shown in the web at right. So far in this chapter, you have considered how functions and their inverses are related in different representations including equations, tables, and graphs. What would the inverse for each of the parent functions you worked with in Chapter 2 look like in each representation?



As you work with your team today, ask each other these questions:

What does the parent function look like in this representation?

How can that help us visualize the inverse?

Would another representation be more helpful?

How can we describe the relationship in words?

5-52. So far, you have worked with various parent functions:

i. $y = x^2$ *ii.* $y = x^3$ *iii.* $y = x$ *iv.* $y = |x|$

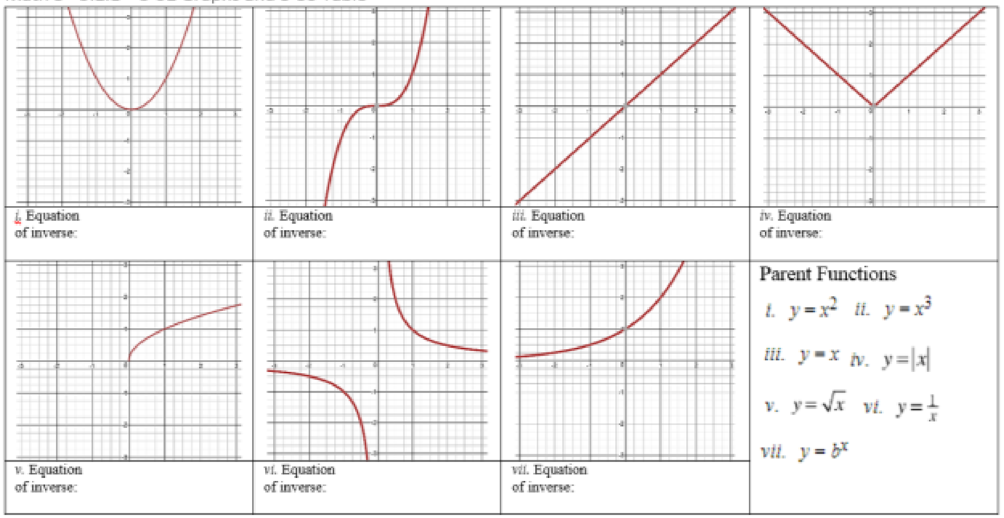
v. $y = \sqrt{x}$ *vi.* $y = \frac{1}{x}$ *vii.* $y = b^x$

a. Sketch a graph of each parent function and then sketch a graph of its inverse. For each parent function, write equation of its inverse, if possible. For this problem the inverse does not have to be a function.

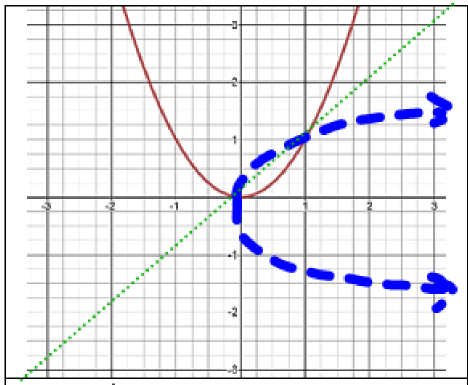
b. Are any parent functions their own inverses? Explain how you know.

c. Do any parent functions have inverse graphs that are not functions? If so, which ones?

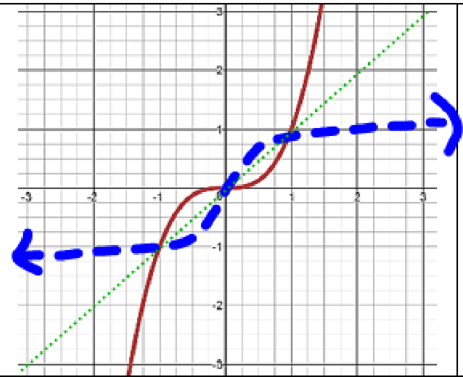
Math 3 - 5.2.1 – 5-52 Graphs and 5-53 Table



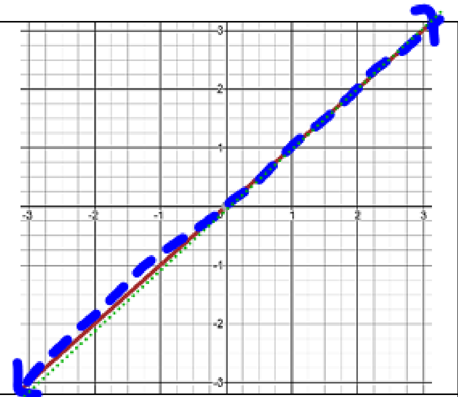
Math 3 - 5.2.1 – 5-52 Graphs and 5-53 Table



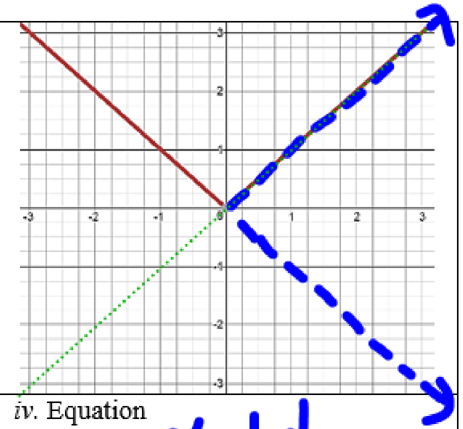
i. Equation of inverse: $y=\pm\sqrt{x}$



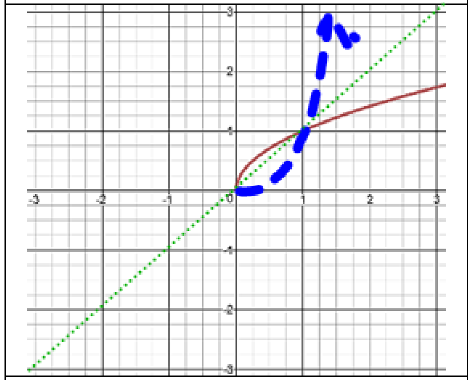
ii. Equation of inverse: $y=\sqrt[3]{x}$



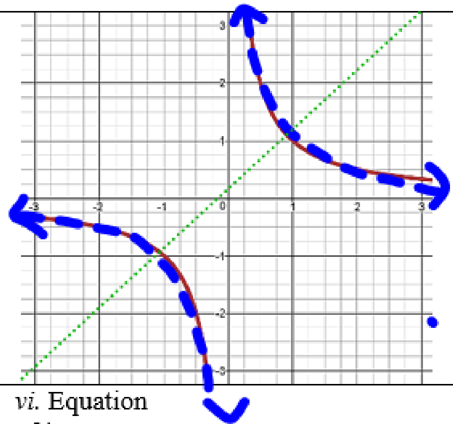
iii. Equation of inverse: $y=x$



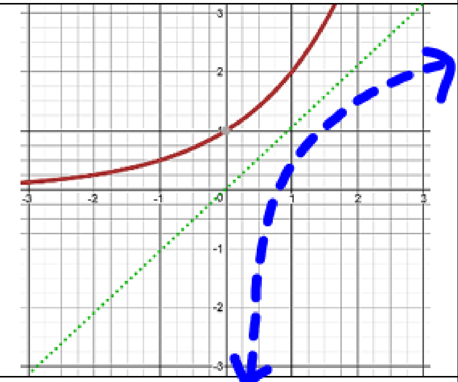
iv. Equation of inverse: $x=|y|$



v. Equation of inverse: $y=x^2$ $\{x\geq 0\}$



vi. Equation of inverse: $y=\frac{1}{x}$



vii. Equation of inverse: ?

Parent Functions

i. $y=x^2$ *ii.* $y=x^3$

iii. $y=x$ *iv.* $y=|x|$

v. $y=\sqrt{x}$ *vi.* $y=\frac{1}{x}$

vii. $y=b^x$

$y(x)=\left(\frac{1}{y}\right)^y$

$\frac{yx}{x}=\frac{1}{x} \rightarrow y=\frac{1}{x}$

5-53. THE INVERSE EXPONENTIAL FUNCTION

The function $y = b^x$ has an inverse function, but you do not yet know how to write its equation in $y =$ form. Since exponential functions are useful for modeling everyday situations, the inverse of an exponential function is also important. Use $y = 3^x$ as an example. Even though you may not know how to write the equation of the inverse function of $y = 3^x$ in $y =$ form, you already know a lot about it.

a. Make a table for the inverse of $y = 3^x$.

b. Sketch a graph of the inverse of $y = 3^x$.

c. If the input for the inverse function is 81, what is the output? Explain your reasoning.

d. Using your answers from parts (a) through (c), if you input any number for x into the inverse function, how can you describe the output?

$y = 3^x$

x	−3	−2	−1	0	1	2
y	$\frac{1}{27}$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9

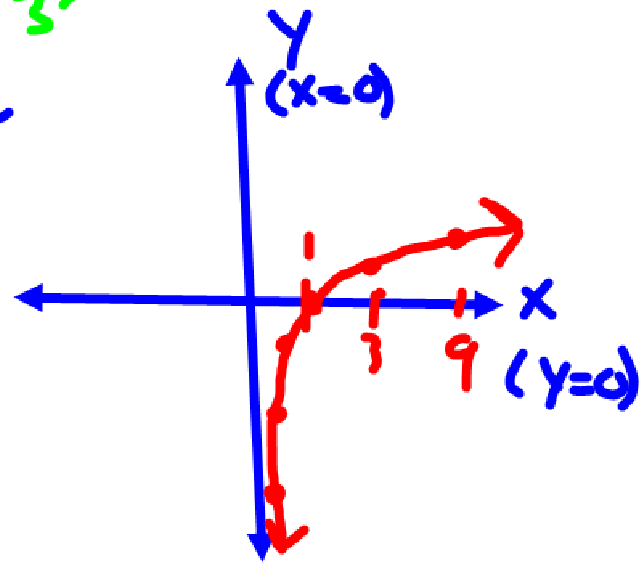
$y = 3^{-3}$

$y = \frac{1}{3^3}$

Inverse

x	$\frac{1}{27}$	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9	81
y	−3	−2	−1	0	1	2	4

Graph



5-54. AN ANCIENT PUZZLE

Parts (a) through (f) below are similar to a puzzle that is more than 2100 years old. Mathematicians first created the puzzle in ancient India in the 2^{nd} century BCE. More recently, about 700 years ago, Muslim mathematicians created the first tables allowing them to locate answers to this type of puzzle quickly. Tables similar to them appeared in school math books until recently.

Here are some clues to help you figure out how the puzzle works:

$\log_2(8) = 3$ $\log_3(27) = 3$

$\log_5(25) = 2$ $\log_{10}(10{,}000) = 4$

Use the clues to determine the missing pieces of the puzzles below:

a. $\log_2(16) = ?$ b. $\log_2(32) = ?$ c. $\log_?(100) = 2$

d. $\log_5(?) = 3$ e. $\log_?(81) = 4$ f. $\log_{100}(10) = ?$

5-55. How is the Ancient Puzzle related to the inverse function for $y = 3^x$ in problem 5-53? Show how you can use the idea in the Ancient Puzzle to write an equation in $y =$ form for the inverse function of $y = 3^x$.