

Write the inverse
(in words):

Add 7

Multiply by 3

Divide by 2

Square Root

x	y
1	3
2	6
3	9

$$\begin{array}{r} 3 \overline{) 1} \\ 9 \\ \hline 3 \\ \hline 0 \end{array}$$

$$\left[\begin{array}{l} \text{Square} \\ \cdot 2 \\ \div 3 \\ - 7 \end{array} \right.$$

Write the inverse:

$$f(x) = 3x - 6$$

$$f^{-1}(x)$$

x	y
4	6
5	9

$$f^{-1}(x) = \frac{x+6}{3}$$

$$\cancel{\frac{x}{3} + 6}$$

Why did the
Scarecrow win
an award?

Write the inverse:

$$p(x) = 2(x + 3)^3$$

$$p^{-1}(x) = \sqrt[3]{\frac{x}{2}} - 3$$

+3
cube
÷2

÷2

cube root

-3

$$\frac{1}{2} \overline{) 128}$$

$$\sqrt[3]{128}$$

Write the inverse:

$$t(x) = \frac{10(6-4)}{3} \cdot$$

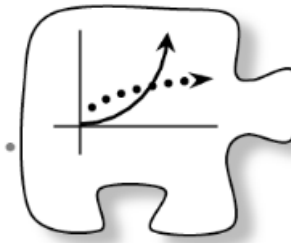
$$\begin{array}{l} -4 \\ \cdot 10 \\ \div 3 \end{array}$$

Inverse

$$\begin{array}{l} \cdot 3 \\ \div 10 \\ + 4 \end{array}$$

$$t^{-1}(x) = \frac{3x}{10} + 4$$

5.1.2 How can I determine an inverse?



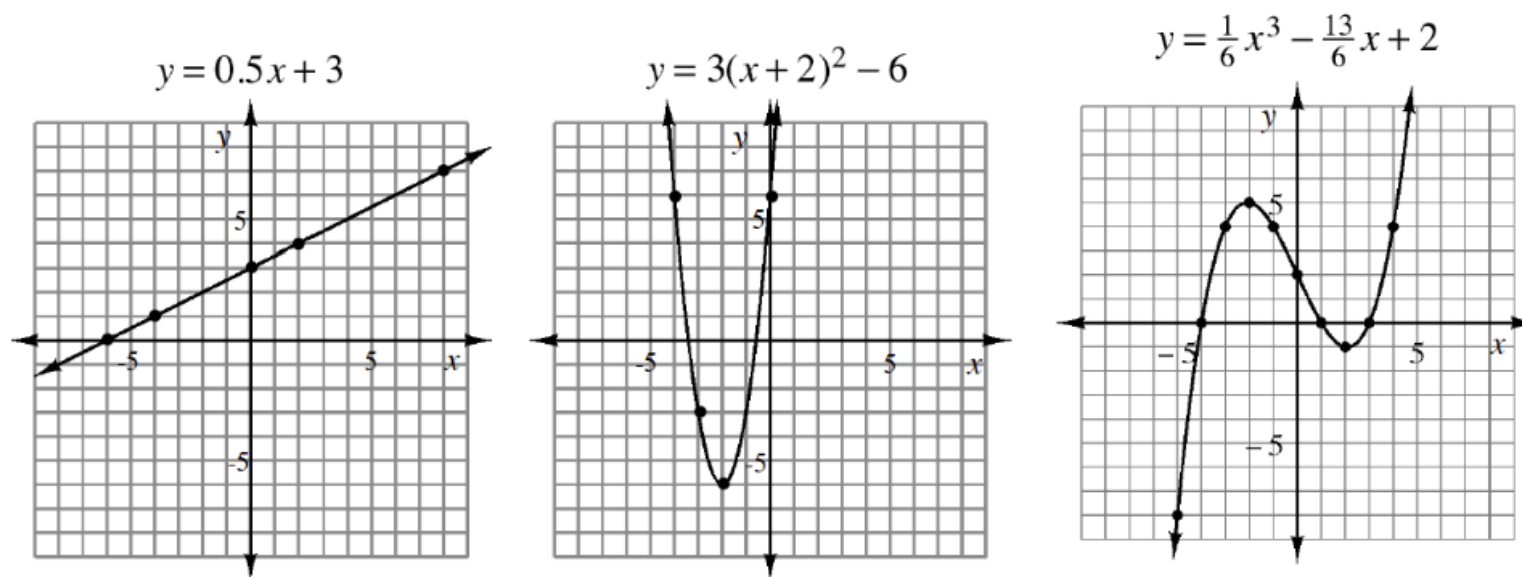
Using a Graph to Determine an Inverse

What things would you consider if you were thinking about buying a car? The first things that come to mind might be color and cost, but increasingly people are considering fuel efficiency (the number of miles a car can drive on a gallon of gas). You can think of the average number of miles per gallon that a car gets as a function that has *gallons* as the input and *miles traveled* as the output. A graph of this function would allow you to use what you know about the number of gallons in your tank to predict how far you could travel.



However, suppose you are driving a route where there are many miles between gas stations. In this scenario, you might want to figure out how many gallons of gas you will need to get to the next gas station. In this case, you would start with the number of miles and work backwards to solve for gallons. Your new function would reverse the process.

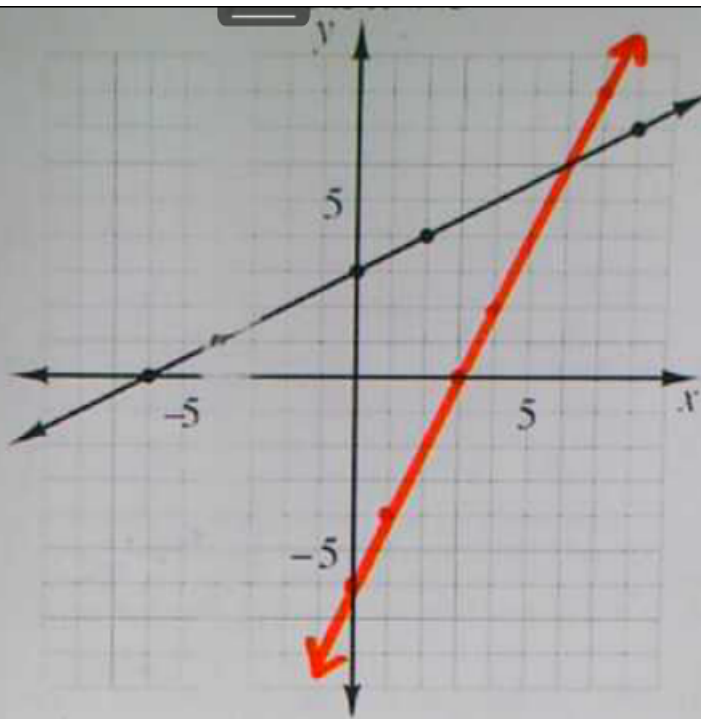
5-15. In Lesson 5.1.1 you started with functions and worked backwards to determine the equations of their inverses. Now you will focus on functions and their inverses represented as graphs. Use what you discovered yesterday as a basis for completing the problems below.



- Obtain a [Lesson 5.1.2 Resource Page](#) and make a careful graph of each function's inverse on the same set of axes as its corresponding function. Look for a way to make the graph without determining the equation of the inverse first. Be prepared to share your strategy with the class.
- Describe the relationship between the coordinates of a function and the coordinates of its inverse. Use tables of the function and its inverse to show what you mean.

$$f(x)$$

x	y
-6	0
-4	1
0	3
2	4
8	7



$$f^{-1}(x)$$

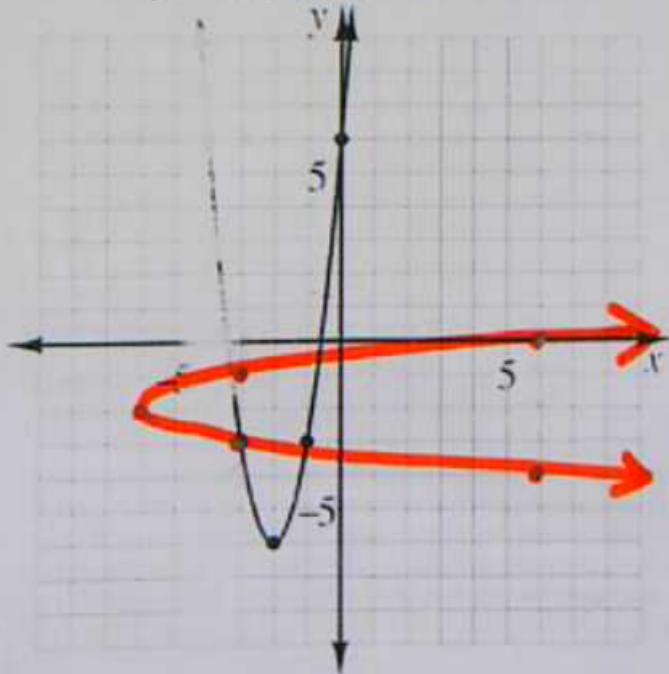
x	y
0	-6
1	-4
3	0
4	2
7	8

11) $-8 - 7 = -15$

$$f(x)$$

x	y
0	0
-1	-3
-2	-6
-3	-3
-4	0

$$y = 3(x + 2)^2 - 6$$



$$f^{-1}(x)$$

x	y
0	0
-3	-1
-6	-2
-3	-3
0	-4

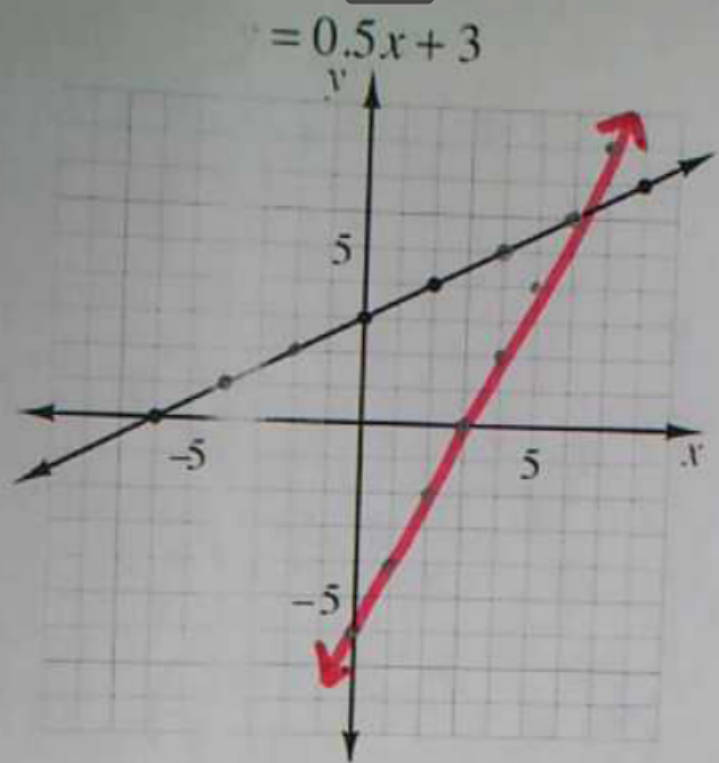
12) $\frac{-20}{-2} = 10$

SELMA HIGH SCHOOL

MONDAY	TUESDAY
Aug 13	14
Every day	I
and the pp	(10)

$f(x)$

x	y
-6	0
-4	1
-2	2
0	3
2	4
4	5

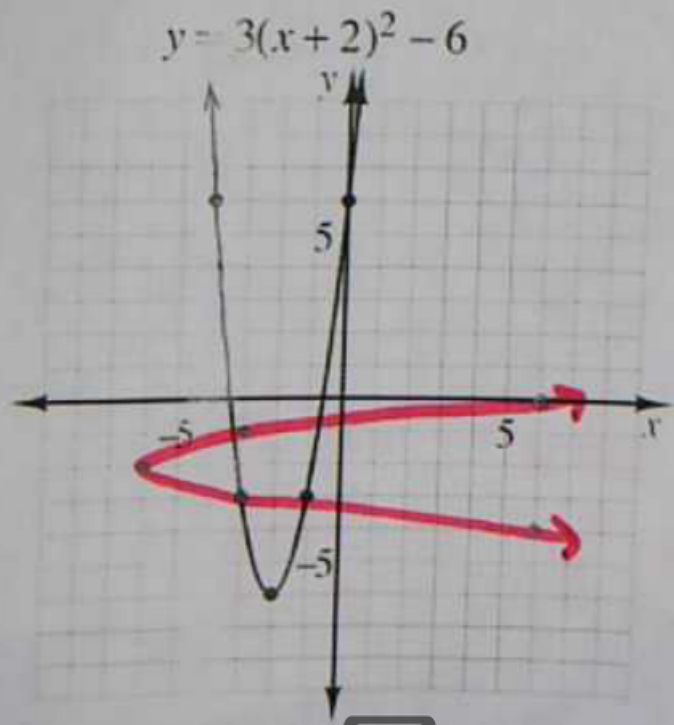


$f^{-1}(x)$

x	y
0	-6
1	-4
2	-2
3	0
4	2
5	4

$f(x)$

x	y
-4	6
-3	3
-2	-6
-1	-3
0	6



$f^{-1}(x)$

x	y
6	-4
-3	-3
-6	-2
-3	-1
6	0

11) $-8 \cdot 4 = -32$
 13) $\frac{12}{-6} = -2$

10) $-8 \cdot 8 = -64$
 12) $\frac{-20}{-2} = 10$

Round each to the place indicated.

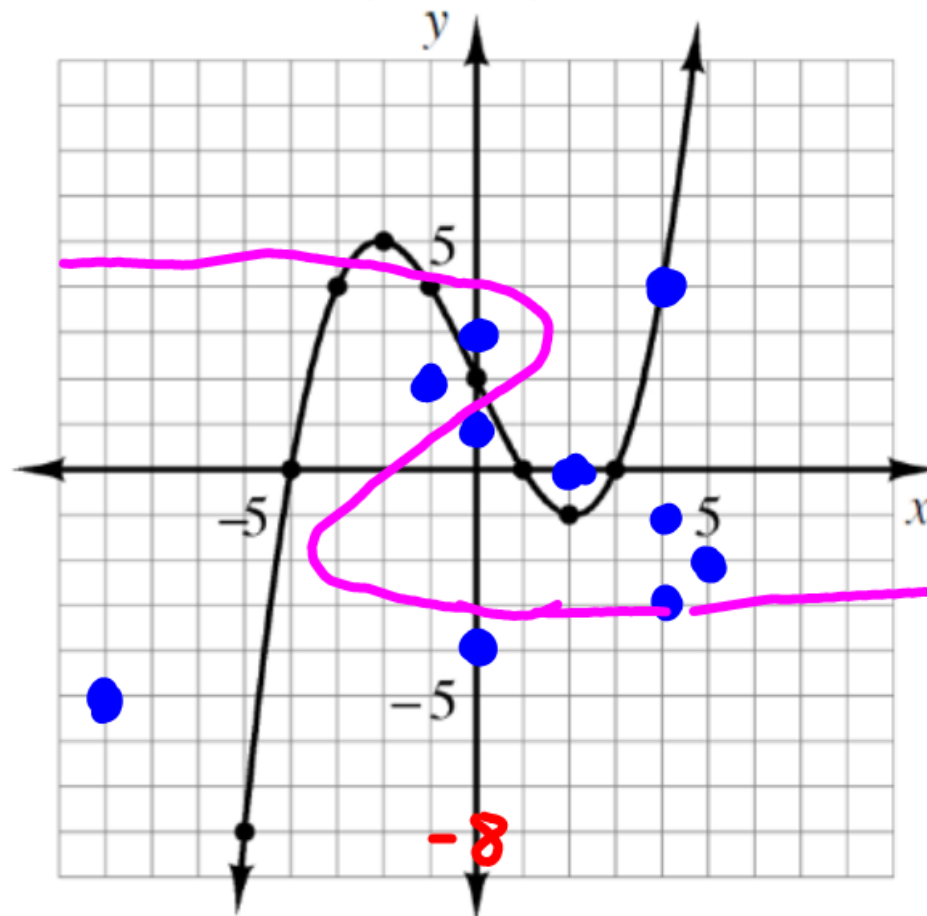


SELM
 HIGH
 MONDAY

$f(x)$

x	y
-5	-8
-4	0
-3	4
-2	5
-1	4
0	2
1	0
2	-1
3	0
4	4

$$y = \frac{1}{6}x^3 - \frac{13}{6}x + 2$$

 $f^{-1}(x)$

x	y
-8	-5
-5	-4
-4	-3
-3	-2
-2	-1
-1	0
0	1
1	2
2	0
3	4
4	4

5-16. When you look at the graph of a function and its inverse, you can see a symmetrical relationship between the two graphs demonstrated by a line of symmetry.

a. Draw the line of symmetry for each graph in problem 5-15.

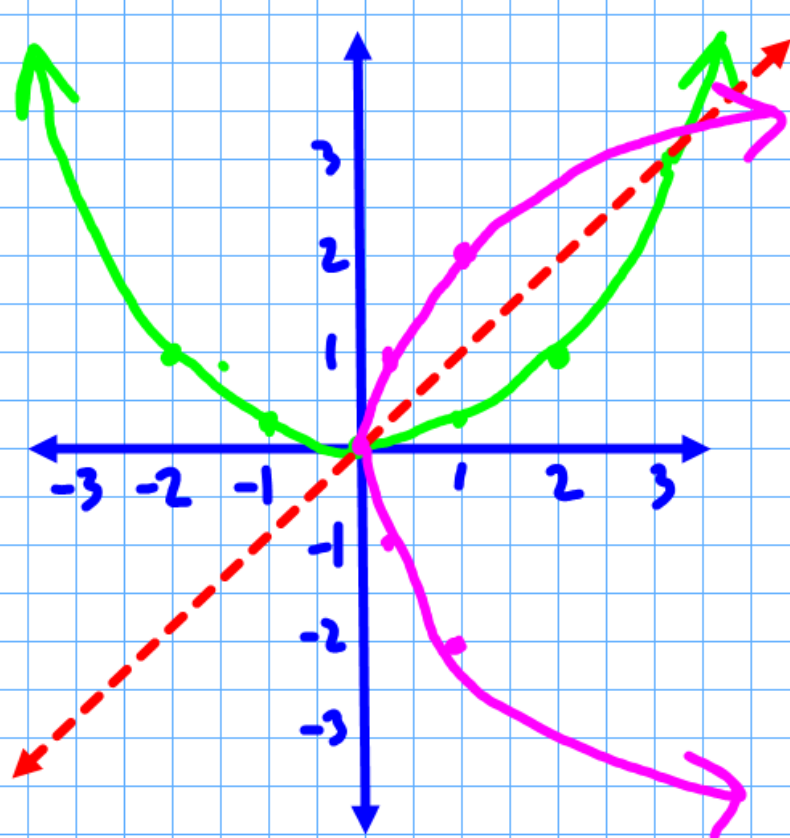
b. What is the equation of the line of symmetry for each graph?

$$y=x$$

c. Why do you think this line makes sense as the line of symmetry between the graph of a function and its inverse?

5-17. You can use the line of symmetry you identified in problem 5-16 to help graph the inverse of a function without creating a table.

- Make a complete graph of $y = \left(\frac{x}{2}\right)^2$ on a full sheet of graph paper. Use the same scale for the x - and y -axes.
- On the same set of axes, graph the line of symmetry $y = x$.



x	$f(x) = \left(\frac{x}{2}\right)^2$
-2	1
-1	.25
0	0
1	.25
2	1

Inverse.	
x	y
1	-2
.25	-1
0	0
.25	1
1	2

5-18. Now you will look at the relationship between the domain and range of a function and its inverse.

- What are the domain and range of $y = \left(\frac{x}{2}\right)^2$? What are the domain and range of its inverse?
- How are the domain and range of the original function related to the domain and range of the inverse?

5-19. Your graphing calculator can also help you to graph the inverse of a function. Use the inverse-drawing feature of your graphing calculator to verify that your inverse graph from problem 5-17 is correct. [Draw Inverse eTool](#) (Desmos) Click in the lower right corner of the graph to view it in full-screen mode. [Desmos Accessibility](#)

$$y = \left(\frac{x}{2}\right)^2$$

D: all real #'s; $(-\infty, \infty)$; $-\infty < x < \infty$

R: $y \geq 0$; $0 \leq y < \infty$; $[0, \infty)$

Inverse:

D: $x \geq 0$ R: $-\infty < y < \infty$

5-20. Write the equation of the inverse of $y = \left(\frac{x}{2}\right)^2$. Is there another way to write it? If so, show how the two equations are equivalent. Demonstrate that your inverse equation undoes the original function and use a graphing calculator to check the graphs.

$$y = \left(\frac{x}{2}\right)^2$$

÷2

Square

Inverse

√

·2

$$y = \sqrt{x} \cdot 2$$

$$y = -\sqrt{x} \cdot 2$$

$$\sqrt{x^2} = \sqrt{4}$$

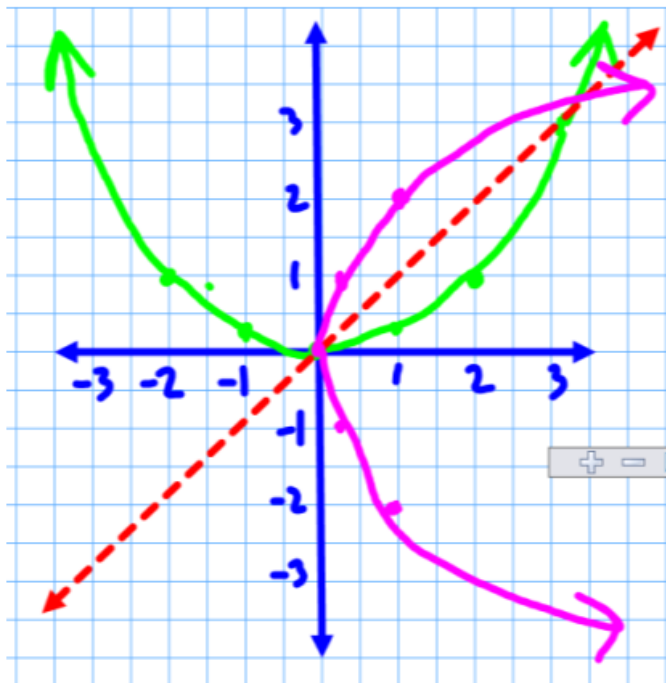
$$|x| = 2$$

$$\swarrow \quad \searrow$$

$$x = 2 \quad x = -2$$

5-21. Consider your equation for the inverse of $y = \left(\frac{x}{2}\right)^2$.

- Is the inverse a function? How can you tell?
- Use color to trace over the portion of your graph of $y = \left(\frac{x}{2}\right)^2$ for which $x \geq 0$. Then use another color to trace the corresponding part of the inverse graph. Write an equation for the traced portion of the inverse graph.
- In part (b) you **restricted the domain** of the original function so that the inverse was also a function. Is there a different way to restrict the domain so that the inverse is a function? What is the equation of this inverse function?



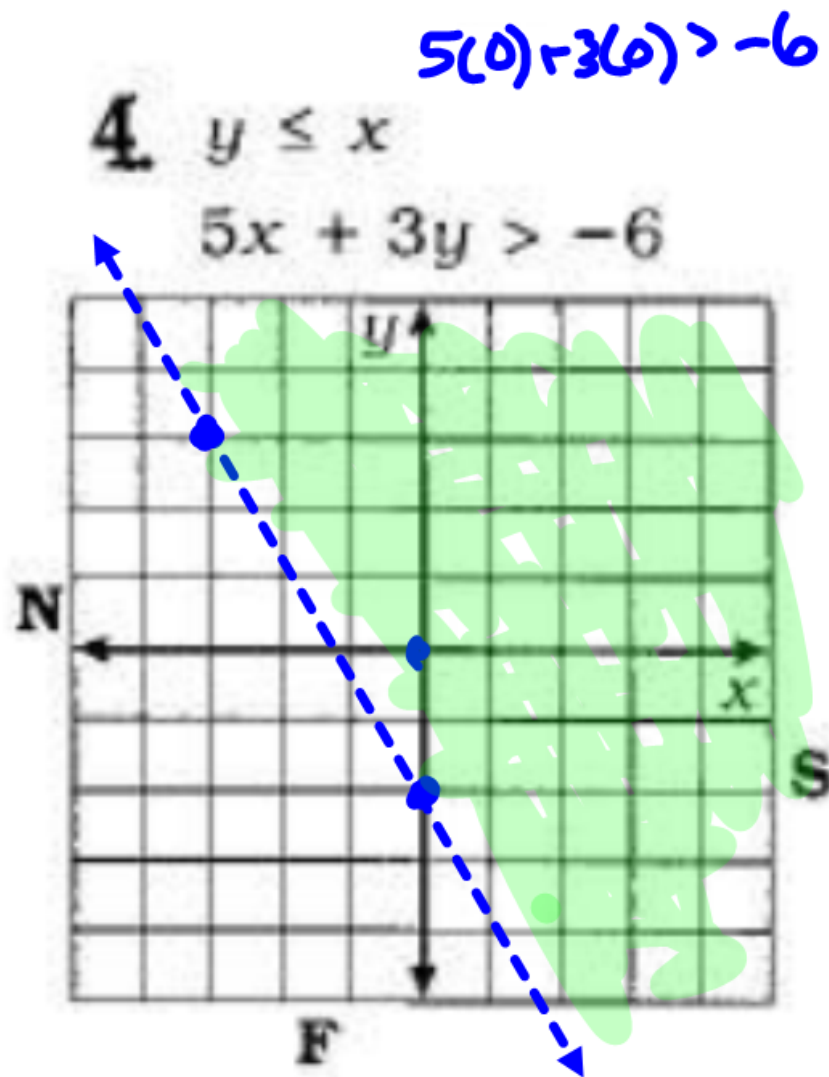
5-22. In problem 5-21, you restricted the domain of the function $y = \left(\frac{x}{2}\right)^2$ to $x \geq 0$ so that its inverse would be a function.

Now consider the function $g(x) = (x - 3)^2$.

- How can you restrict the domain of $g(x)$ so that its inverse will be a function?
- Graph $y = g(x)$ with its domain restricted, and then graph $y = g^{-1}(x)$ on the same set of axes.
- What is the equation for $g^{-1}(x)$?

$$\{x \geq \underline{3}\}$$

$$g^{-1}(x) = \sqrt{x} + 3$$



$$y = mx + b$$

$$5x + 3y = -6$$

$$-5x \quad -5x$$

$$\frac{3y}{3} = \frac{-5x - 6}{3}$$

$$y = -\frac{5}{3}x - 2$$

5-23. How can you determine from a graph whether its inverse will be a function? Explain. What are some examples of other functions whose inverses are not functions?

5-24. Graph $y = 1 + \frac{2}{x}$ and its inverse on the same set of axes. Label the graph and the inverse function with their equations.

- a. What is the equation of the inverse graph? Is it a function?
- b. What are the domain and range of both the original function and the inverse?