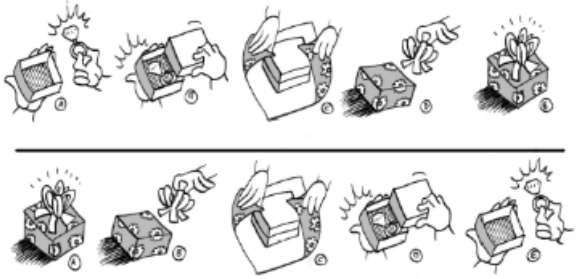
# 5.1.1 How can I "undo" a function?

## "Undo" Equations

Have you ever heard someone say, "*She knows it forward and backward*," to describe someone who understands an idea deeply? Often, being able to reverse a process is a way to show how thoroughly you understand it. Today you will reverse mathematical processes, including the inner workings of functions. As you work today, keep these questions in mind:

How can we "undo" it?

How can we justify each step?



**5-1.** GUESS MY NUMBER

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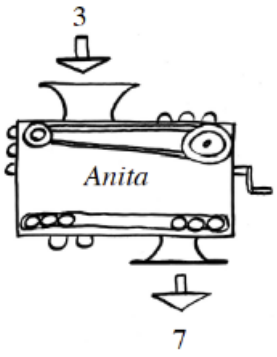
Today you will play the "Guess My Number" game. Your teacher will think of a number and tell you some information about that number. You will try to determine your teacher's number. (You can use your calculator or pencil and paper if it helps.) When you think you know the number, sit silently and do not tell anyone! Be sure to give others a chance to figure it out!

For example your teacher might say: *"When I add 4 to my number and then multiply the sum by 10, I get –70. What is my number?"*

Your task will be to calculate the number and explain your reasoning.

**5-2.** A picture of Anita's function machine is shown at right. When she puts 3 into the machine, 7 comes out. When she puts 4 in, 9 comes out, and when she puts –3 in, –5 comes out.




a. Make a table to organize the inputs and outputs from Anita's function machine. Explain in words what this machine is doing to the input to generate an output.

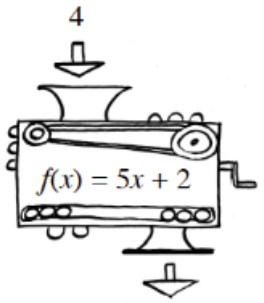
b. Anita's function machine suddenly starts working backwards: it begins pulling outputs back up into the machine, reversing the machine's process, and returning the original input. If 7 is pulled back into this machine, what value do you think will come out of the top?

c. Anita sets up her new backwards function machine and enters the other outputs. What would you expect to come out the top if 9 is entered? If –5 is entered? Explain.

d. Record the inputs and outputs of the backwards function machine in a table. Record the numbers being pulled back in as $x$, and the numbers coming out the top as $y$. Explain in words what Anita's backwards function machine is doing.

e. Write equations for Anita's original function machine and for her backwards machine. How are the two functions related?

**5-3.** The function machine at right uses the function $f(x) = 5x + 2$.




a. If the crank is turned backwards, what number should be pulled up into the machine to have a 4 come out of the top?

b. Keiko wants to build a new machine that will undo what $f(x)$ does to an input. What must Keiko's machine do to 17 to undo $f(x)$ and return a value of 3? ① subt. 2 ② divide 5

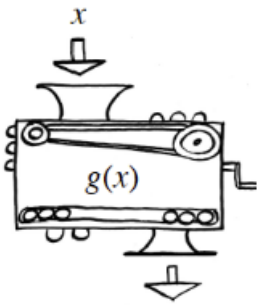
c. An "undo" function is called an **inverse function** and has the notation $f^{-1}(x)$. Note that the –1 is not a negative exponent. It is the mathematical symbol that indicates the inverse function of $f(x)$. Write an equation for $f^{-1}(x)$, Keiko's "undo" function machine.

d. Choose a value for $x$. What strategy can you use to show that your equation, $f^{-1}(x)$, undoes the effects of the original function machine $f(x)$? Explain.

$f(x) = +5x+2$ $\qquad$ $f^{-1}(x) = \frac{x-2}{5}$

$\sin(\theta) = \frac{1}{2}$ $\qquad$ $\sin^{-1}\left(\frac{1}{2}\right) = \theta$

**5-4.** Keiko is working with a new function, $g(x)$. She writes down the following steps for $g(x)$:




- Add 5.
- Divide by 2.
- Cube it.
- Multiply by 6.

Inverse:
÷6
$\sqrt[3]{}$
·2
−5

a. What is the equation for $g(x)$? What is the output when 3 is the input?

b. Help Keiko write down the steps (in words) for the inverse machine, $g^{-1}(x)$, and then write its equation.

c. Verify that your equation in part (b) correctly "undoes" the output of $g(x)$ in part (a).

$g(x) = 6\left(\frac{x+5}{2}\right)^3$

$g^{-1}(x) = \sqrt[3]{\frac{x}{6}}\,2 - 5$

$g^{-1}(x) = 2\sqrt[3]{\frac{x}{6}} - 5$

**5-5.** What are the inverse functions for each of the functions below? Use function notation. Justify that each equation for the inverse works.

a. $f(x) = 3x - 6$

b. $g(x) = x^3 - 5$

c. $p(x) = 2(x + 3)^3$

d. $t(x) = \frac{10(x-4)}{3}$

**5-6.** Choose one function and its inverse from the previous problem. Make sure each team member has a different function. Then make a complete graph and a table for your function and its inverse. Graph the function and its inverse on the same set of axes.

When each person in your team has finished, put everyone's work into the middle of the workspace. Describe what relationships you see between the representations of a function and its inverse.

**5-7.** LEARNING LOG

What strategies did your team use to write equations for inverses of functions? How can you be sure that the equations you wrote are correct? Discuss this idea and then write a Learning Log entry about the strategies you have for writing the equations of inverses of functions and for checking that they work. Title this entry "Writing and Checking the Equation of an Inverse Function" and include today's date.