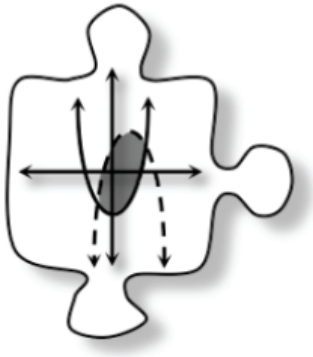
3.2.1 How can I solve inequalities?



Solving Inequalities with One or Two Variables

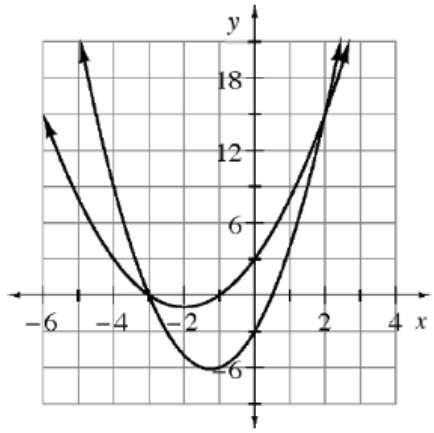
In this chapter, you have developed many strategies for solving equations with one variable and solving systems of equations with two variables. But what if you want to solve an inequality or system of inequalities instead? Today you will explore how to use familiar strategies to solve inequalities. As you work, the questions below can help focus team discussions:

What strategy should we use?

How do we know if this solution is correct?

How can we be sure we have found all of the solutions?

3-67. In the previous section, you learned how to use the graph of a system to solve an equation. How can the graphs of $y = 2x^2 + 5x - 3$ and $y = x^2 + 4x + 3$ (shown at right) help you solve an *inequality*? Consider this as you complete the parts below.



a. How can you use the graph to determine the solutions to $2x^2 + 5x - 3 = x^2 + 4x + 3$? What are the solutions?

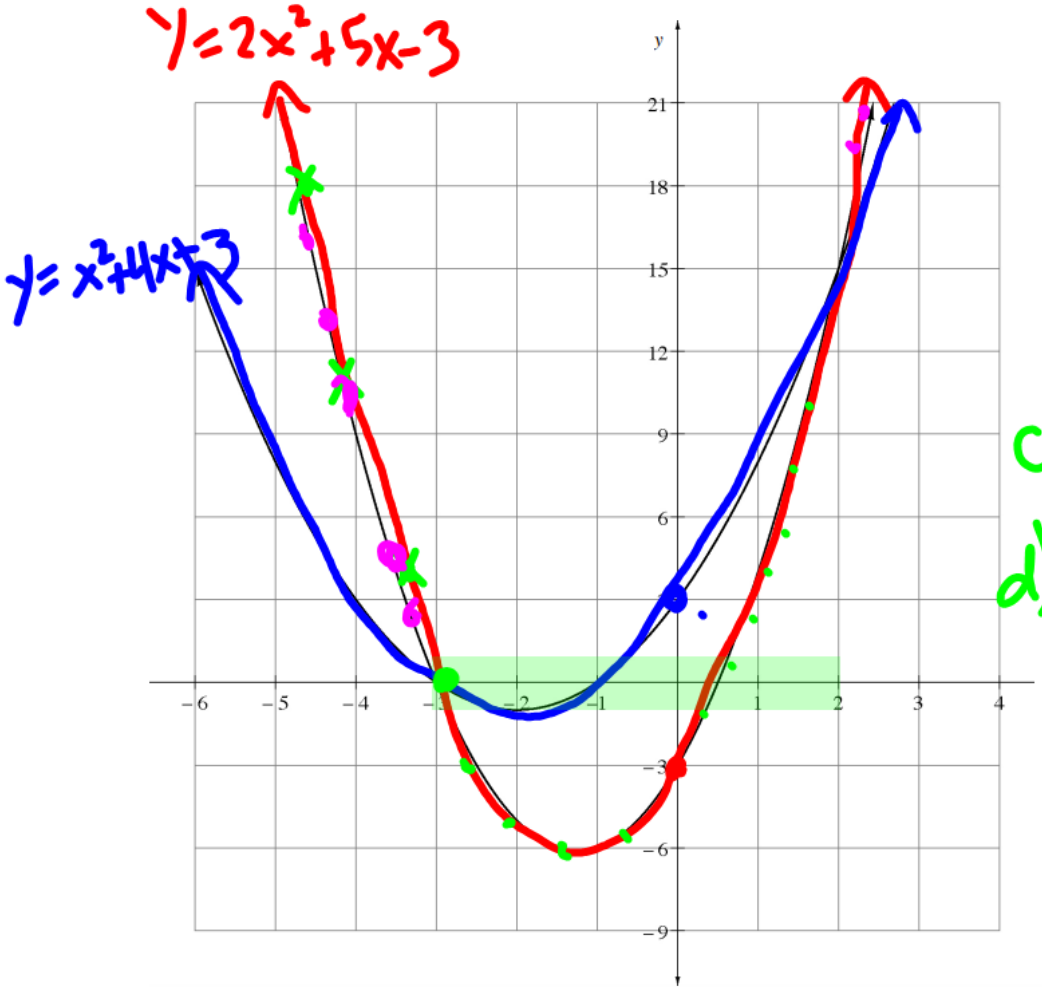
b. Obtain a Lesson 3.2.1A Resource Page or use the *3-67 Student eTool* (Desmos). On the resource page, label each graph with its equation and highlight each function with a different color. How did you decide which graph matches which function? Click in

$y = 2x^2 + 5x - 3$

$y = x^2 + 4x + 3$

$2x^2 + 5x - 3 = x^2 + 4x + 3$

$x = -3 \quad x = 2$

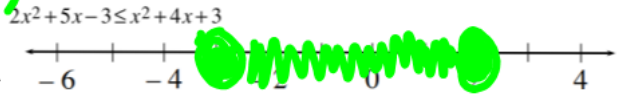


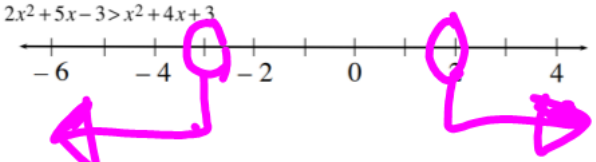
$y = 2x^2 + 5x - 3$

$y = x^2 + 4x + 3$

c) $-3 \leq x \leq 2$

d)





$x < -3$ or $x > 2$

c. Use the graph to identify the x-values for which $2x^2 + 5x - 3 \leq x^2 + 4x + 3$. How did you locate the solutions? How many solutions are there? How can you describe all of the solutions?

d. How can these solutions be represented on a number line? Locate the number line labeled $2x^2 + 5x - 3 \leq x^2 + 4x + 3$ on your resource page. Use a colored marker to highlight the solutions to the inequality on the number line.

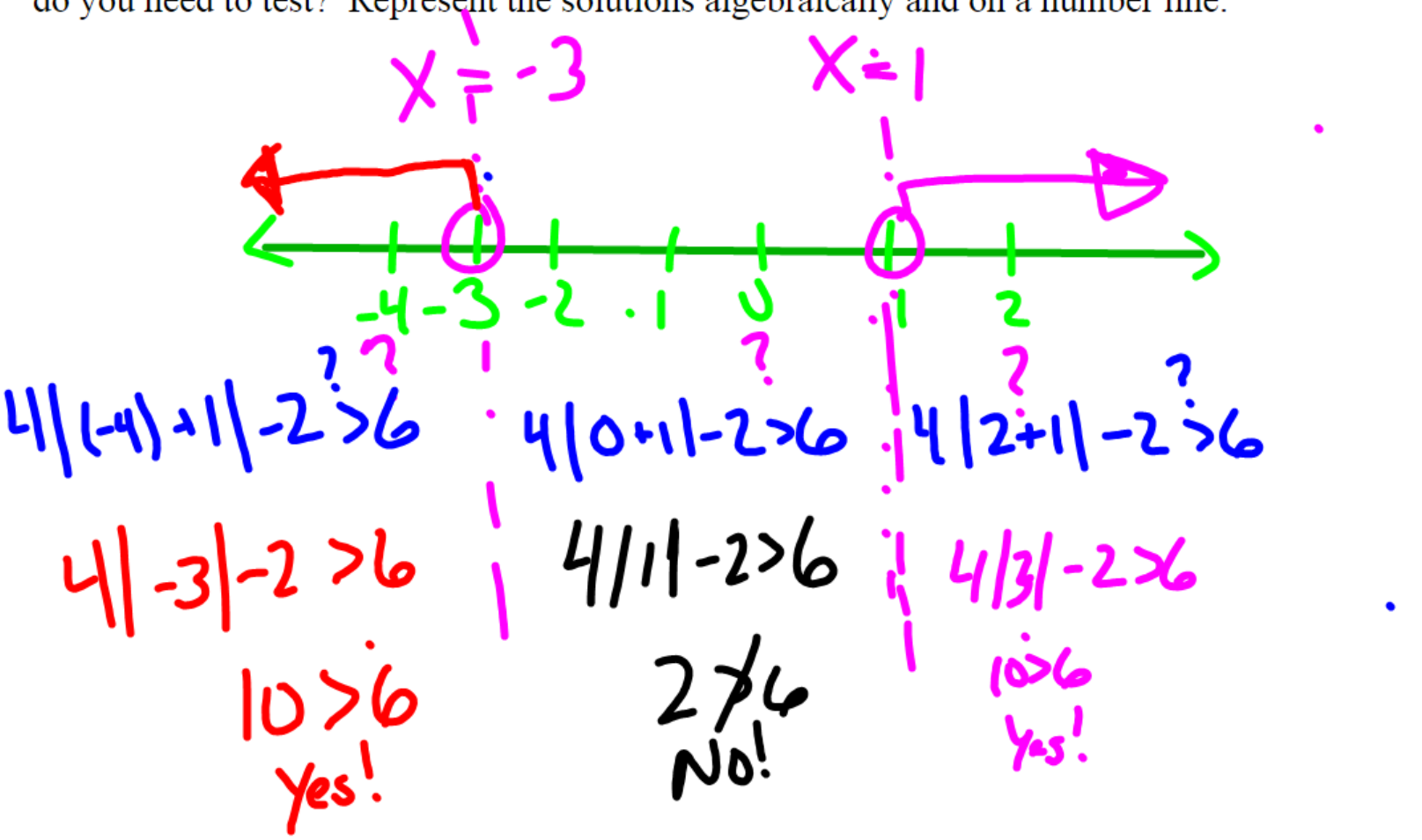
e. What about the inequality $2x^2 + 5x - 3 > x^2 + 4x + 3$? What are the solutions to this inequality? Represent your solutions algebraically and on a number line.

3-68. Consider the inequality $4|x+1|-2>6$.

$x<-3$ or $x>1$

a. How many boundary points are there? What are they? Should they be marked with filled or unfilled circles? Mark the boundary points on a number line.

b. Which portion(s) of the number line contain the solutions for this inequality? How many regions do you need to test? Represent the solutions algebraically and on a number line.



3-69. Bert and Ernie are solving the inequality $2x^2 + 5x - 3 < x^2 + 4x + 3$. They are looking at the graph in problem 3 67 when Bert has an idea. "*Can't we change this into one parabola and solve our inequality that way?*", he asks.

Ernie asks, "*What do you mean?*"

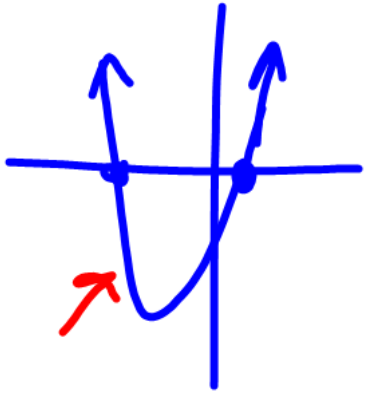
"*Can't we determine the solutions by looking at the graph of* $f(x) = x^2 + x - 6$*?*", Bert replies.

a. Where does Bert get the equation $f(x) = x^2 + x - 6$?

b. Try Bert's idea. Make a sketch of the parabola and show how it can be used to solve the original inequality.

c. *"Just a minute!"* mumbles Ernie, *"I think I have another method. Instead of graphing the parabola, can't we just rewrite the new inequality as* $x^2 + x - 6 < 0$ *and then solve the equation* $x^2 + x - 6 = 0$*? This would give us the boundary points and then we could test numbers in the original inequality to see the regions that contain the solutions."* Use Ernie's method to solve the inequality. Does it give the same solution?

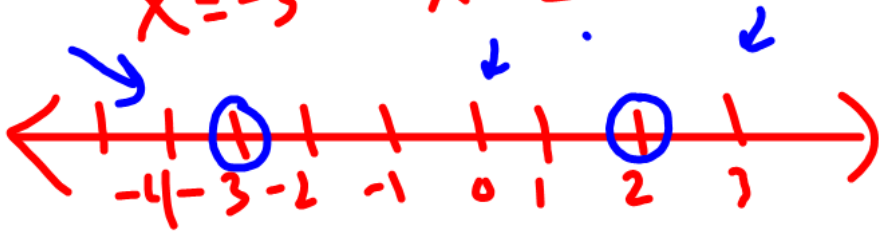
d. Use any method to solve the inequality $x^2 - 3x - 10 \geq 0$.



$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x = -3 \quad x = 2$



3-70. Next, Bert and Ernie work on solving the inequality $4|x+1|-2>6$ from problem 3-68. This time, Ernie had an idea. *"Why don't we solve this by graphing a system of equations as we did in problem 3-67?"*.

a. What system of equations should they graph?

b. Make a complete graph of the system and explain how you can use it to solve $4|x+1|-2>6$.

3-71. In problems 3-67 and 3-69 you looked at solutions to an inequality with one variable (x). Now consider the system of inequalities with two variables (x and y) below.

$$y \geq 2x^2 + 5x - 3$$
$$y < x^2 + 4x + 3$$

a. Which points make both inequalities true? For example, does the point (–3, 0) make both inequalities true? What about (–1, 1)? (1, 5)? Refer to your Lesson 3.2.1A Resource Page to help you answer these questions.

b. What is the difference between a solution to the *system* of inequalities above and a solution to the inequality in problem 3-67?

c. How are the graphs of the equations $y = 2x^2 + 5x - 3$ and $y = x^2 + 4x + 3$ related to the graph of the system of inequalities?

d. With your team, how can you represent all of the solutions to the system of inequalities? Show the solution to the system of inequalities on the Lesson 3.2.1A Resource Page.