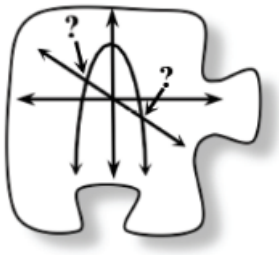
# 3.1.3 How many solutions are there?




## Multiple Solutions to Systems of Equations

You have used many different algebraic and graphical strategies to solve equations with one variable. You have also worked with systems of equations with two variables. In this lesson, you will use your algebraic strategies and graphing tools to determine the number of solutions that various systems of equations have and to determine the meaning of those solutions.

**3-45.** Solve each system of equations algebraically. For each one, explain what the solution (or lack thereof) tells you about the graph of the system.

a. $y = -3x + 5$
$y = -3x - 1$

b. $y = \frac{1}{2}x^2 + 1$
$y = 2x - 1$

c. $y^2 = x$
$y = x - 2$

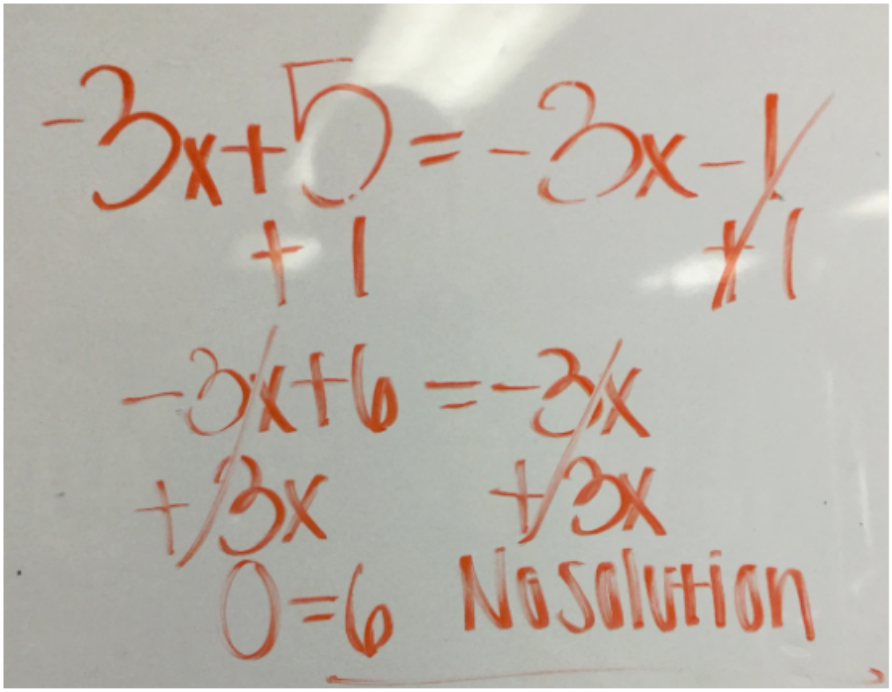
d. $4x - 2y = 10$
$y = 2x - 5$

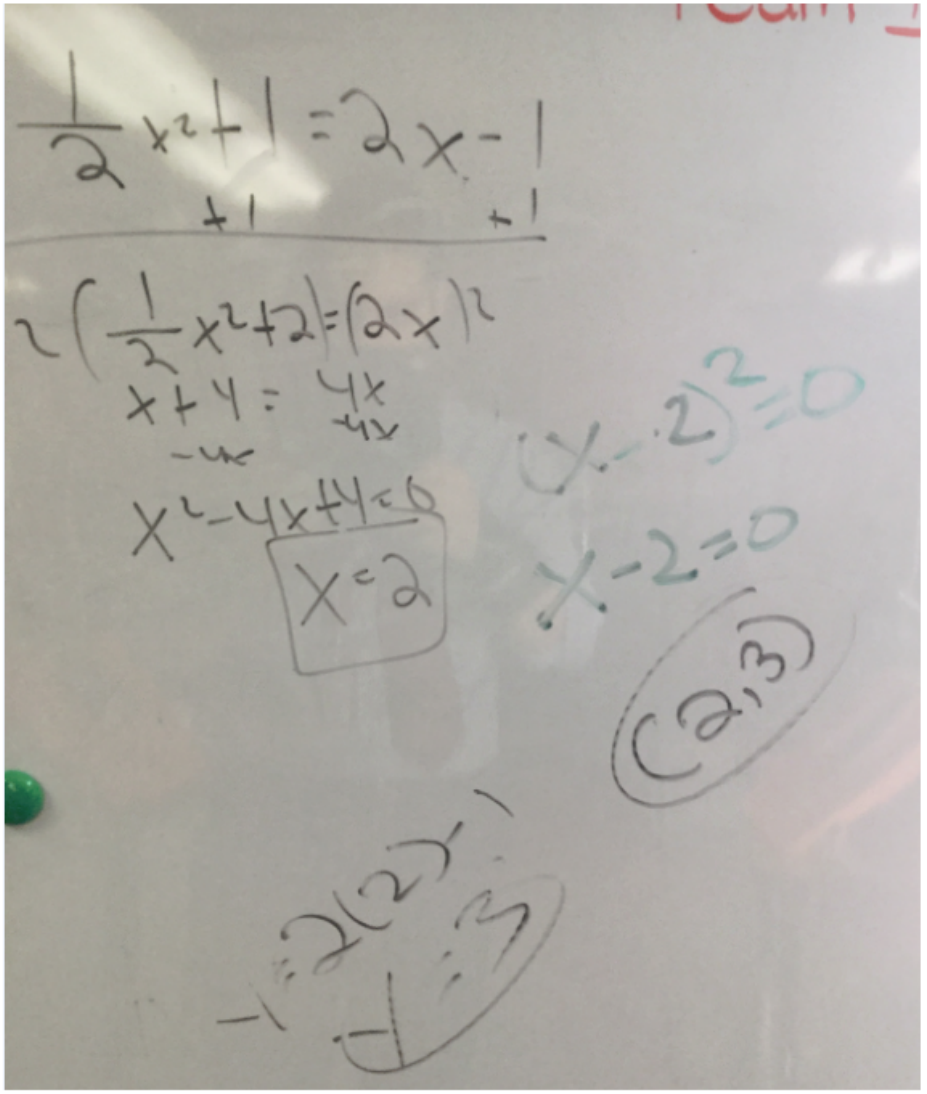
$2x - 1 = \frac{1}{2}x^2 + 1$

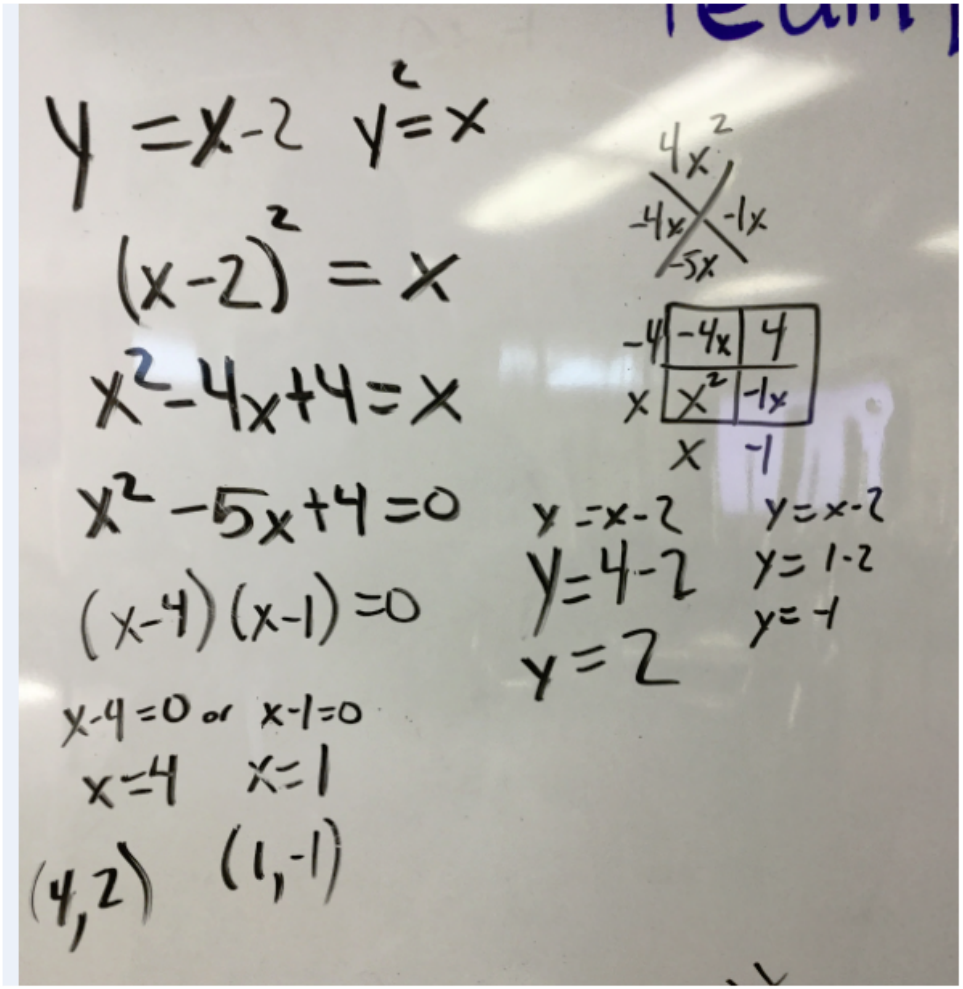
$0 = \frac{1}{2}x^2 - 2x + 2$

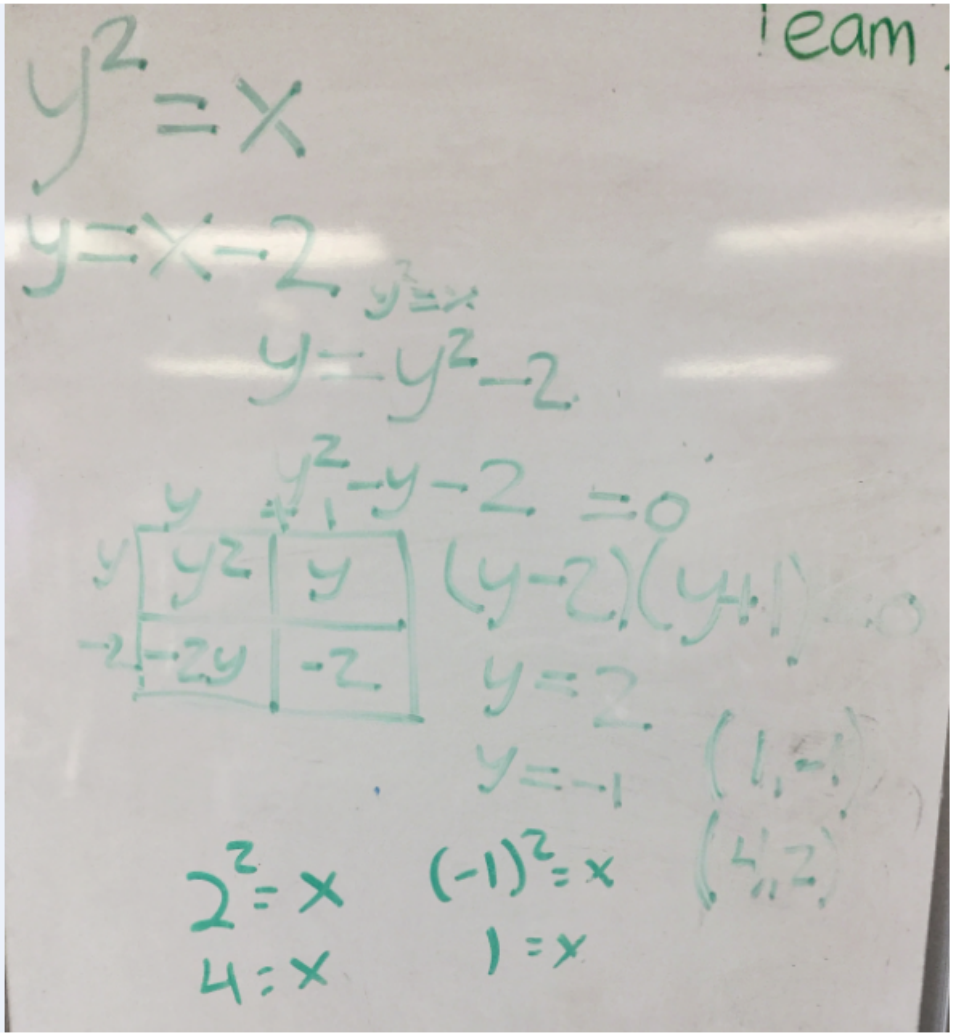
$0 = x^2 - 4x + 4$

$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(.5)2}}{2(\frac{1}{2})}$


-3x+5=-3x-1
+1 +1
-3x+6=-3x
+3x +3x
0=6 No solution


1/2 x² + 1 = 2x - 1
+1 +1
2(1/2 x² + 2 = (2x)²
x² + 4 = 4x
-4x -4x
x² - 4x + 4 = 0
x = 2
(x-2)² = 0
x-2=0
(2,3)
-1=2(2)-1
-1=3


y = x - 2
y² = x
(x-2)² = x
x² - 4x + 4 = x
x² - 5x + 4 = 0
(x-4)(x-1) = 0
x-4=0 or x-1=0
x=4 x=1
(4,2) (1,-1)
4x²
-4x -1x
-5x
-4 -4x 4
x x² -1x
x -1
y = x-2
y = 4-2
y = 2
y = x-2
y = 1-2
y = -1


y² = x
y = x - 2
y = y² - 2
y² - y - 2 = 0
y -y
y y² -y
-2 -2y -2
(y-2)(y+1) = 0
y = 2
y = -1
(1,-1)
(4,2)
2² = x
4 = x
(-1)² = x
1 = x

**3-46.** Now consider the system shown below:

$$x^2 + y^2 = 25$$

$$y = x^2 - 13$$

a. How many solutions do you expect this system to have? Explain how you made your prediction.

b. Solve this system by graphing. How many solutions do you see? Was your prediction in part (a) correct?

c. Combine these equations to create a new equation so that the only variable is $x$. Then combine the equations in a different way to create a new equation that contains only the variable $y$. Which of these equations would be easier to solve? Why?

d. If you have not already done so, solve one of your equations from part (c). If solving becomes too difficult, you may want to switch to the other combined equation.

**3-47.** In problem 3-46, you solved the system of equations shown below. Explore using the *3-47 Student eTool* (Desmos).
Desmos Accessibility

$x^2 + y^2 = 25$

$y = x^2 - 13$

a. What minor adjustments can you make to an equation (or both equations) in this system so that the new system has n real solutions? Have each member of your team find a different way to alter the system. Justify algebraically that your system has no real solutions. Also, be ready to share your strategies for changing the system along with your justification with the class.

b. Work with your team to alter the system three more times so that the new systems have three, two, and one solution(s). For each new system that your team creates, solve the system algebraically to study how the algebraic solution helps indicate how many solutions the system has. Be prepared to explain what different situations occur during solving that result in different numbers of solutions.