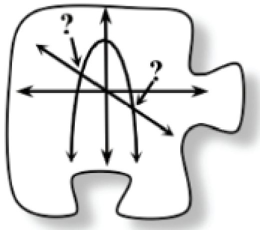
3.1.1 How can I solve the equation?



Strategies for Solving Equations

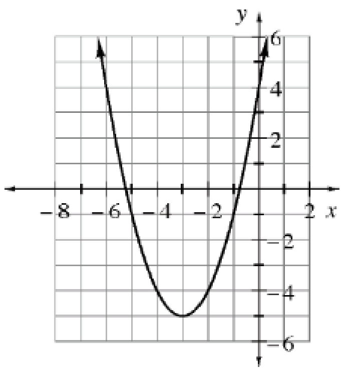
Today you will have the opportunity to solve some challenging equations. As you work with your team, you will be challenged to use multiple approaches and to write clear explanations to show your understanding. As you work, use the questions below to keep your team's discussion productive and focused.

How can we make it simpler?

Does anyone see another way?

How can we be sure the equations are equivalent?

3-1. SOLVING GRAPHICALLY



One of the big ideas of Chapter 2 was how to determine special points on the graph of a function. For example, you used the equation of a parabola written in graphing form to locate its vertex without graphing. But what about the locations of other points on the parabola? Consider the graph of $y = (x + 3)^2 - 5$ at right. Explore the graph using the *3-1 Student eTool* (Desmos). Click in the lower right corner of the graph to view it in full-screen mode. Desmos Accessibility

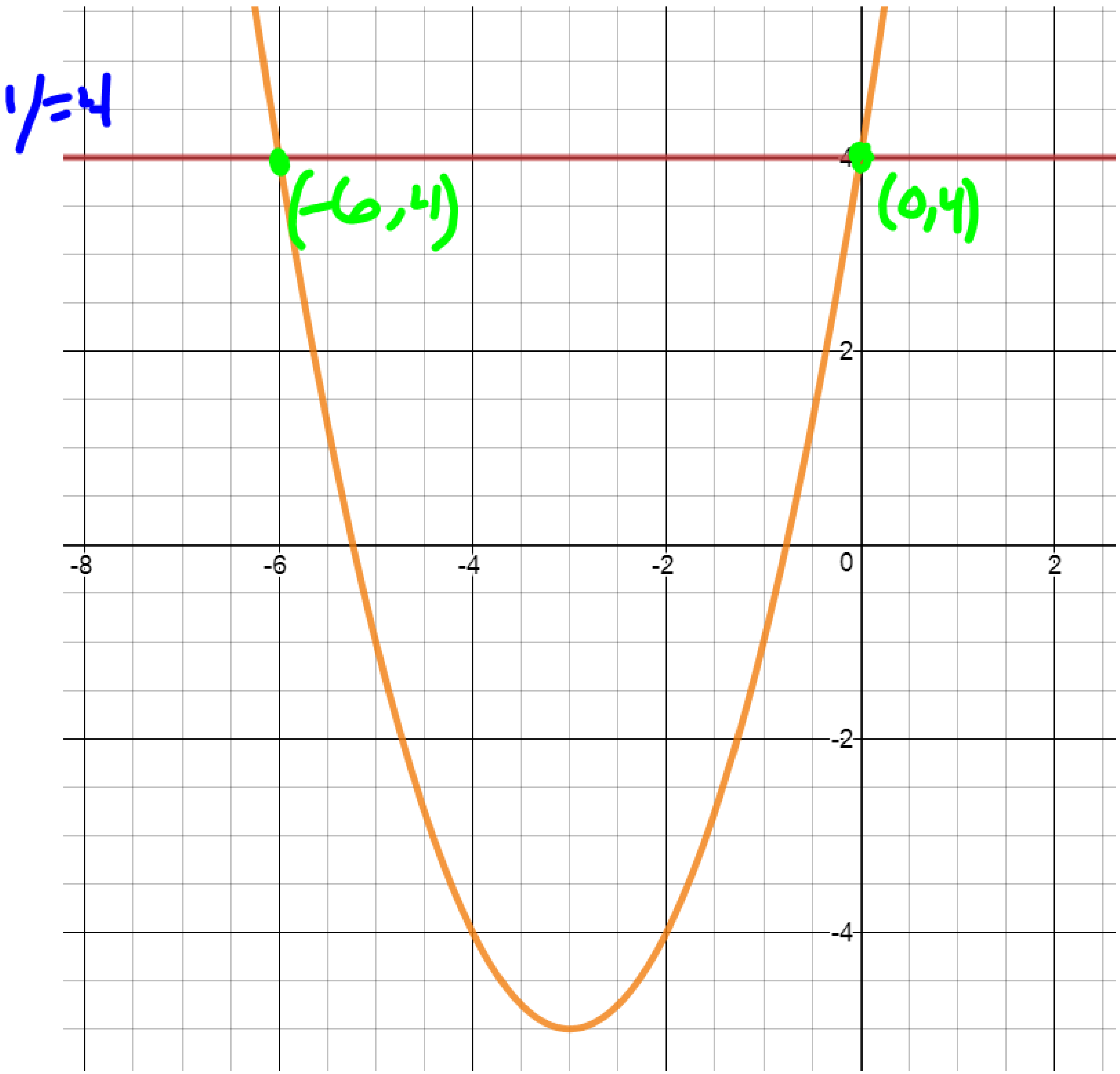
a. How many solutions does the equation $y = (x + 3)^2 - 5$ have? How is this shown on the graph?

b. How many solutions does the equation $(x + 3)^2 - 5 = 4$ have? How is this shown on the graph?

c. Use the graph to solve the equation $(x + 3)^2 - 5 = 4$. How did the graph help you solve the equation? Be sure to discuss this with your team before recording your answer.

$y = (x-3)^2 - 5$

$4 = (x-3)^2 - 5$



3-2. ALGEBRAIC STRATEGIES

The graph in problem 3-1 was useful to solve an equation like $(x + 3)^2 - 5 = 4$. But what if you do not have an accurate graph? And what can you do when the solution is not on a grid point or is off your graph? In these cases, algebraic strategies are useful.

Your Task: Solve the equation below in at least three different ways. The "Discussion Points" below are provided to help you get started. Be ready to share your strategies with the class.

i. $(x + 3)^2 - 5 = 11$ *ii.* $(x + 3)^2 - 5 = 3$

Discussion Points

What algebraic strategies might be useful?

What makes this equation look challenging? How can we make the equation simpler?

How can we be sure that our strategy helps us determine *all* possible solutions?

$(x+3)^2-5=11$

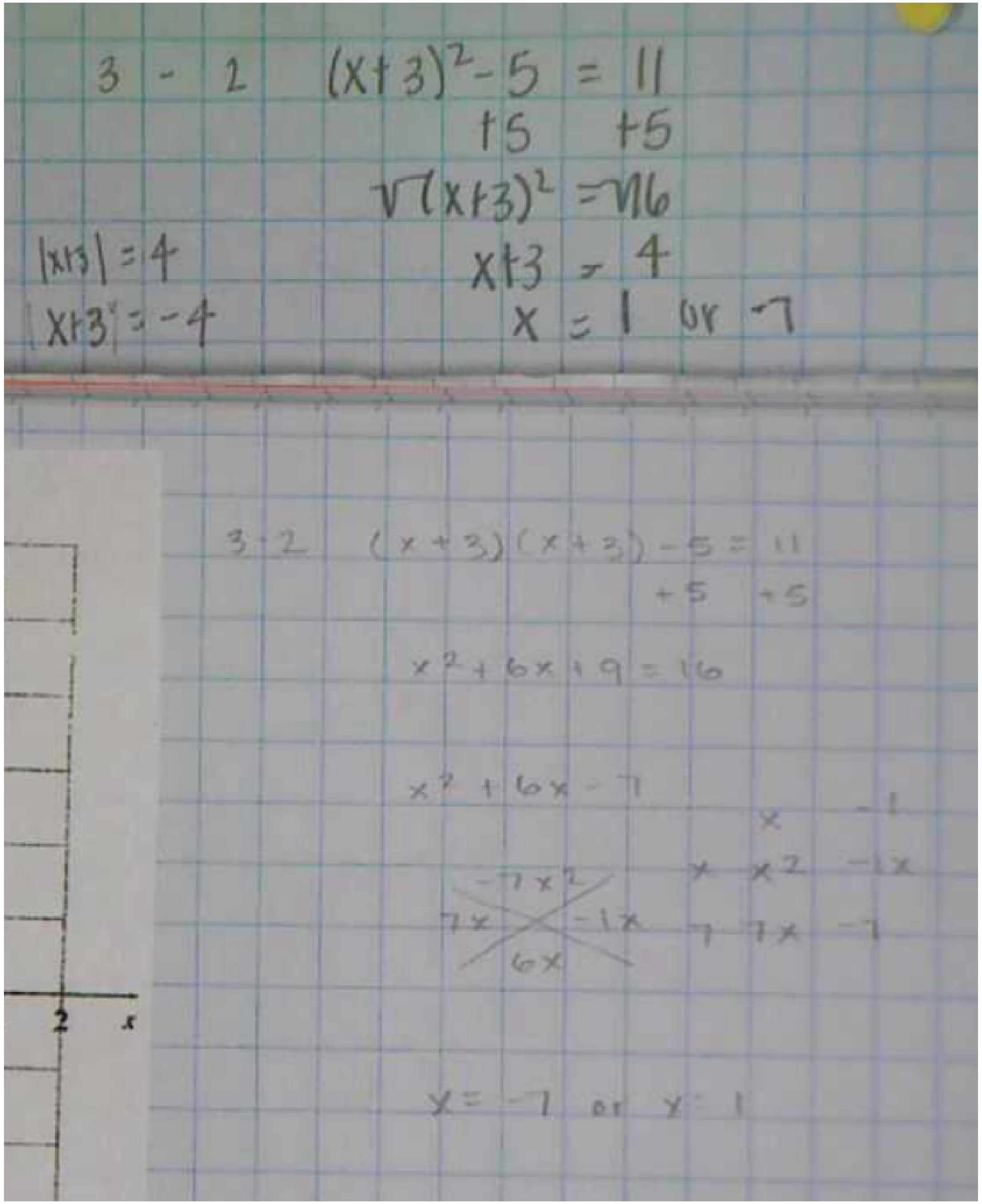
$(x+3)^2=16$

$?^2=16$

$?=4 \qquad ?=-4$

$x+3=4 \qquad x+3=-4$

$x=1 \qquad x=-7$



3-3. Three strategies your class or team may have used in problem 3-2 are:

- **Rewriting**: Using algebra to write a new equivalent equation that is easier to solve.
- **Looking Inside**: Reasoning about the value of the expression inside the function or parentheses.
- **Undoing**: Reversing or doing the opposite of an operation; for example, taking a square root to eliminate squaring.

These strategies and others will be useful throughout the rest of this course. Examine how each of these strategies can be used to solve the equation below by completing parts (a) through (c).

$$\frac{x-5}{4}+\frac{2}{5}=\frac{9}{10}$$

a. Ernie decides to multiply both sides of the equation by 20 so that his equation becomes $5(x-5)+8=18$. Which strategy does Ernie use? How can you tell?

b. Elle takes Ernie's equation and decides to subtract 8 from both sides to get $5(x-5)=10$. Which strategy does Elle use?

c. Eric looks at Elle's equation and says, "*I can tell that (x – 5) must equal 2 because 5 · 2 = 10. Therefore, if x – 5 = 2, then x must be 7.*" What strategy does Eric use?

$$20\left(\frac{x-5}{4}+\frac{2}{5}\right)=\left(\frac{9}{10}\right)20$$

Rewriting $20\left(\frac{x-5}{4}\right)+20\left(\frac{2}{5}\right)=\left(\frac{9}{10}\right)20$

$$5(x-5)+8=18$$

Undoing

$$\begin{array}{r} 5(x-5)+8=18 \\ -8 \quad -8 \\ \hline 5(x-5)=10 \end{array}$$

Looking Instead

$$x-5=2$$

$$x=7$$

3-4. Graciela is trying to solve the quadratic equation $x^2 + 2.5x - 1.5 = 0$. *"I think I need to use the Quadratic Formula because of the decimals,"* she tells Walter. Walter replies, *"I'm sure there's another way! Can't we rewrite this equation so there aren't any decimals?"*

a. What is Walter talking about? Rewrite the equation so that it has no decimals.

b. Rewrite your equation again, so that you can solve it without using the Quadratic Formula. Then solve your equation. Be sure to check your solution(s) with your graphing calculator using Graciela's original equation.

-1	$-1x$	-3
$2x$	$2x^2$	$6x$
	x	$+3$

$$x^2+2.5x-1.5=0$$

$$2x^2+5x-3=0$$

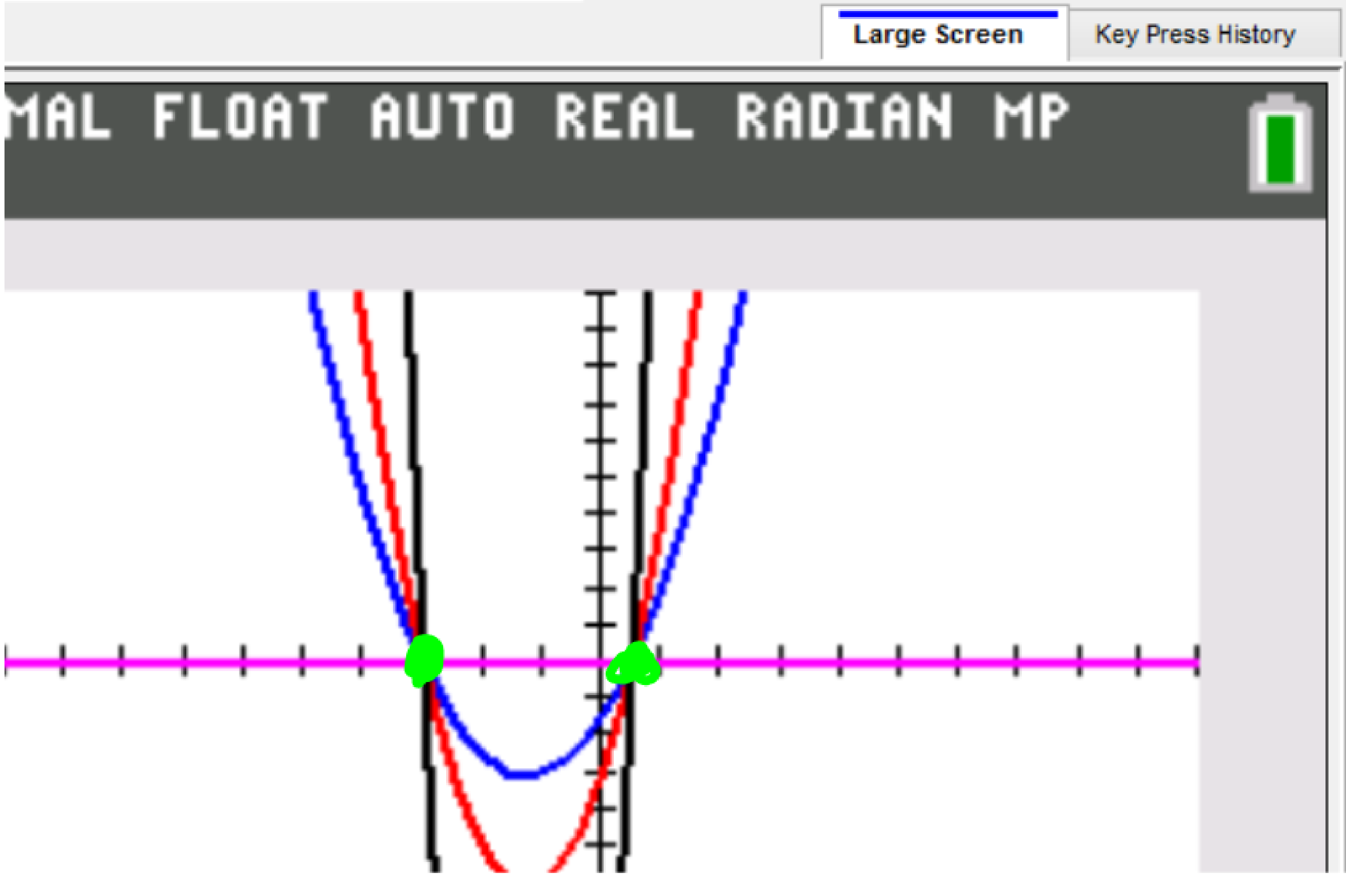
$$10x^2+25x-15=0$$

$$5(2x-1)(x+3)=0$$

$\cancel{\boxed{5=0}}$

$2x-1=0 \quad x+3=0$

$x=0.5 \quad x=-3$



NORMAL FLOAT AUTO REAL RADIAN MP

Plot1 Plot2 Plot3

$Y_1 = X^2 + 2.5X - 1.5$

$Y_2 = 2X^2 + 5X - 3$

$Y_3 = 10X^2 + 25X - 15$

$Y_4 = 0$

$Y_5 =$

$Y_6 =$

$Y_7 =$

Equation

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PRESS + FOR △Tbl

X	Y1	Y2	Y3	Y4
-3.5	2	4	20	0
-3	0	0	0	0
-2.5	-1.5	-3	-15	0
-2	-2.5	-5	-25	0
-1.5	-3	-6	-30	0
-1	-3	-6	-30	0
-.5	-2.5	-5	-25	0
0	-1.5	-3	-15	0
.5	0	0	0	0
1	2	4	20	0
1.5	4.5	9	45	0

X=1.5

Table

3-5. Solve each equation, if possible, using any strategy. Name your strategy and check with your teammates to see what strategies they choose. Be sure to check your solutions algebraically. Then ask your teammates: did you choose the same strategies?

a. $4|8x-2|=8$

b. $3\sqrt{4x-8}+9=15$

c. $(2y-3)(y-2)=-12y+18$

d. $\frac{5}{x}+\frac{1}{3x}=\frac{4x}{3}$

e. $|3-7x|=-6$

f. $\frac{6w-1}{5}-3w=\frac{12w-16}{15}$

g. $(x-3)^2-2=-5$

h. $(x+2)^2+4(x+2)-5=0$

$$(2y-3)(y-2)=-12y+18$$

$$\frac{(2y-3)(y-2)}{(2y-3)}=\frac{-6(2y-3)}{(2y-3)}$$

$$y-2=-6$$

$$y=-4$$