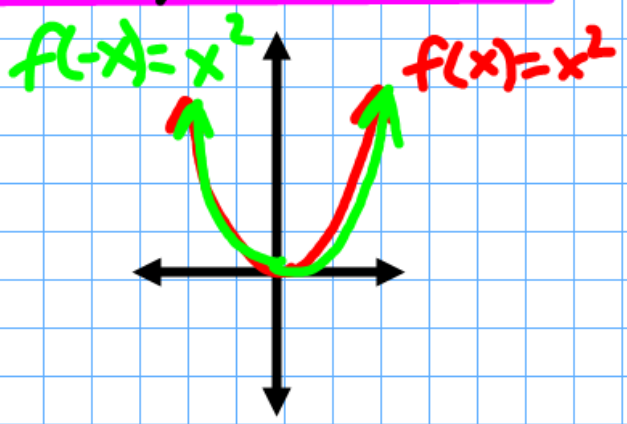
2.2.3 How can I reflect a function?

Transformations of Functions

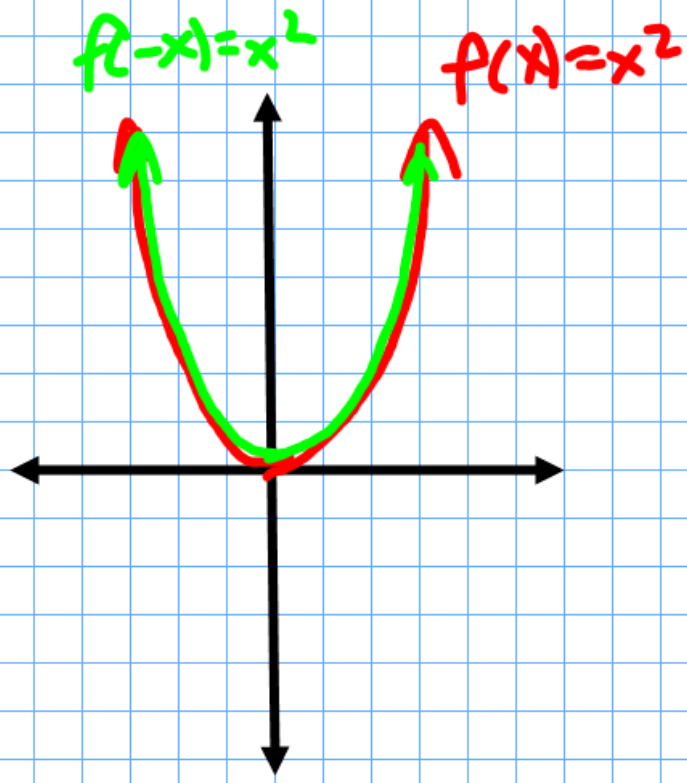
So far in this section, you have looked at how the values of parameters a, h, and k affect the graph of a function $y = af(x - h) + k$. Today you will look at another transformation by exploring what happens when you take the opposite of x before applying the operations of the function. That is, you will investigate $f(-x)$.

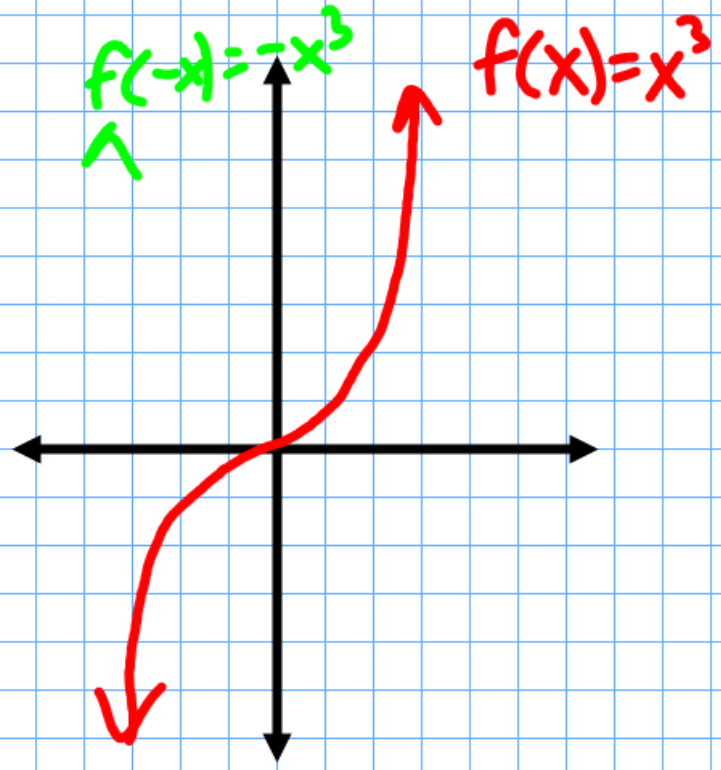
Family	$f(x)$	$f(-x)$
Quadratic	$f(x)=x^2$	$f(-x)=(-x)^2=-x\cdot-x$ $f(-x)=x^2$
Cubic	$f(x)=x^3$	
Abs. Value	$f(x)=\lvert x\rvert$	
Reciprocal	$f(x)=\frac{1}{x}$	
Square Root	$f(x)=\sqrt{x}$	
Exponential	$f(x)=b^x$	

Graph of $f(x)$ and $f(-x)$



Graphs of $f(x)$ and $f(-x)$





2-71. Some functions can be categorized as **even functions** or **odd functions**.

Even functions: All functions where $f(-x) = f(x)$.

Odd functions: All functions where $f(-x) = -f(x)$.

a. For each parent function, write equations for $f(x)$, $f(-x)$, and $-f(x)$.

b. Which of the parent functions are even functions? Which of the parent functions are odd functions?

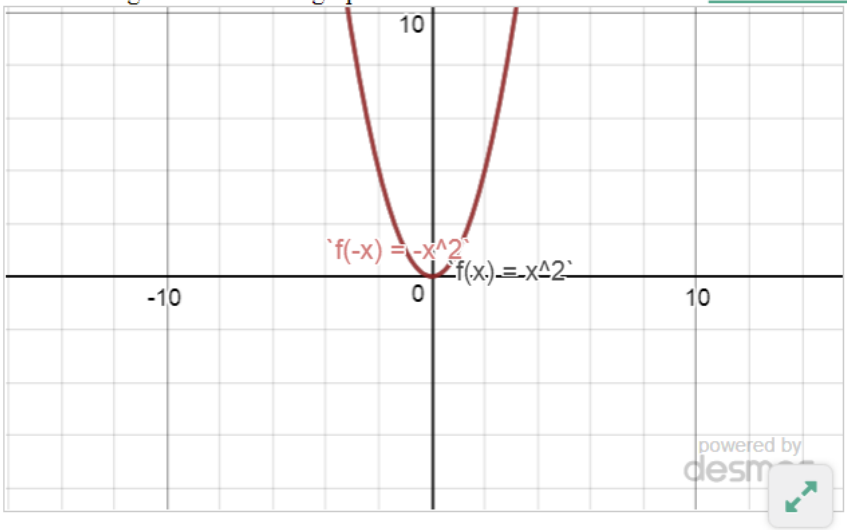
c. How can you determine if a function is even or odd from its graph?

Even functions have reflectional symmetry across the y-axis

Odd functions have rotational symmetry (180º) about the origin

Family	$f(x)$	$f(-x)$	$-f(x)$
Quadratic	$f(x)=x^2$	$f(-x)=(-x)^2=-x\cdot -x$ $f(-x)=x^2$	$-f(x)=-(x^2)$ $-f(x)=-x^2$
Cubic	$f(x)=x^3$	$f(-x)=-x^3$	$-f(x)=-x^3$
Abs. Value	$f(x)=\lvert x\rvert$	$f(-x)=\lvert x\rvert$	$-f(x)=-\lvert x\rvert$
Reciprocal	$f(x)=\frac{1}{x}$	$f(-x)=-\frac{1}{x}=\frac{1}{-x}$	$-f(x)=-\frac{1}{x}$
Square Root	$f(x)=\sqrt{x}$	$f(-x)=\sqrt{-x}$	$-f(x)=-\sqrt{x}$
Exponential	$f(x)=b^x$	$f(-x)=b^{-x}=\frac{1}{b^x}$	$-f(x)=-b^x$

2-70. Investigate the transformation $y = f(-x)$ as directed below. Explore using the *2-70 Student eTool* (Desmos). Click in the lower right corner of the graph to view it in full-screen mode. Desmos Accessibility



a. For each of the parent functions you have investigated so far, write an equation for $f(-x)$ and algebraically simplify the result.

b. For each parent function, draw the graph of the original equation and the new equation on the same set of axes in different colors.

c. Describe your results from part (b). How is the graph of $y = f(x)$ transformed when you replace x with $-x$?

$-x \cdot -x \cdot -x$

$f(x) = x^2$ $\quad f(-x) = x^2$

$f(x) = |x|$ $\quad f(-x) = |-x| = |x|$

$f(x) = b^x$ $\quad f(-x) = b^{-x} = \frac{1}{b^x}$

$f(x) = x^3$ $\quad f(x) = (-x)^3 = -x^3$

$f(x) = \sqrt{x}$ $\quad f(x) = \sqrt{-x}$

$f(x) = \frac{1}{x}$ $\quad f(x) = \frac{1}{-x}$

2-71. Some functions can be categorized as **even functions** or **odd functions**.

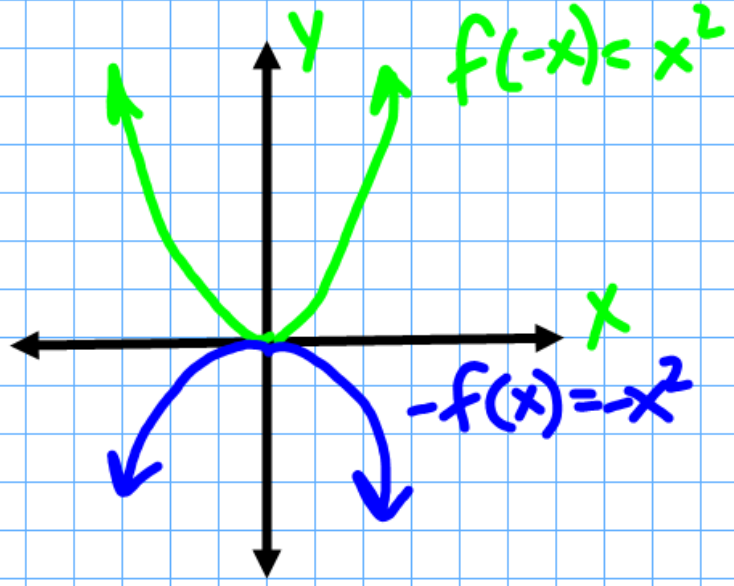
Even functions: All functions where $f(-x) = f(x)$.

Odd functions: All functions where $f(-x) = -f(x)$.

a. For each parent function, write equations for $f(x)$, $f(-x)$, and $-f(x)$.

b. Which of the parent functions are even functions? Which of the parent functions are odd functions?

c. How can you determine if a function is even or odd from its graph?



Quadratic

2-72. GRAPHS OF ODD AND EVEN FUNCTIONS

Your goal in this investigation is to determine whether a function is odd or even by looking at its graph.

a. Use your graphing calculator to graph the following functions, and make a quick sketch of each graph on your paper. Be sure to label each graph.

E N E

$f(x) = x^2$ $f(x) = (x + 5)^2$ $f(x) = x^2 + 5$

O N N

$f(x) = x^3$ $f(x) = (x + 5)^3$ $f(x) = x^3 + 5$

O O

$f(x) = \frac{1}{x}$ $f(x) = -2.5x$

b. Determine which of the functions in part (a) are odd, even, or neither.

c. Classify the function at right as odd, even, or neither. Explain.