2.1.1 What information do I need?

Triangle Congruence Theorems

In Chapter 1, you organized polygons into groups using their attributes and investigated relationships among different polygons. You found that two polygons that look very different, such as a kite and a rectangle, still share the common attribute of having four sides. But what makes two figures look alike?

In today’s lesson, you will identify congruent triangles. As you search for congruent triangles in today’s problems, focus on these questions:

- What do we know about these triangles?
- How can we show they are congruent?
- What are the shortcuts for establishing triangle congruence?

2-1. Recall that if two polygons have the same angle measures and side lengths for all pairs of corresponding angles and sides, then the two polygons are congruent and there is a sequence of rigid transformations that maps one polygon onto the other.

a. Verify that all the corresponding side lengths and all the corresponding angle measures of the two triangles below are equal. What is a sequence of rigid transformations that maps the triangle on the left onto its image on the right? Note: The triangles are drawn to scale.
2.1.1 Triangle Congruence

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a. Verify that all the corresponding side lengths and all the corresponding angle measures of the two triangles below are equal. What is a sequence of rigid transformations that maps the triangle on the left onto its image on the right? Note: The triangles are drawn to scale.

\[
\begin{align*}
\text{m} \angle G + 90^\circ + 28^\circ &= 180^\circ \\
\end{align*}
\]

b. Write a congruence statement for the two triangles. Is there more than one correct congruence statement? Explain.

\[
\begin{align*}
28^\circ &= \text{m} \angle I = \text{m} \angle N \\
90^\circ &= \text{m} \angle R = \text{m} \angle A \\
62^\circ &= \text{m} \angle T = \text{m} \angle G \\
\end{align*}
\]
2.1.1 - Triangle Congruence

Recall that if two polygons have the same angle measures and side lengths for all pairs of corresponding angles and sides, then the two polygons are congruent and there is a sequence of rigid transformations that maps one polygon onto the other.

a. Verify that all the corresponding side lengths and all the corresponding angle measures of the two triangles below are equal. What is a sequence of rigid transformations that maps the triangle on the left onto its image on the right? Note: The triangles are drawn to scale.

\[ \overline{TR} = \overline{GA} \quad m \angle TR = m \angle GA = 160 \text{ units} \]
\[ \overline{RI} = \overline{AN} \quad m \angle RI = m \angle AN = 30 \text{ units} \]
\[ \overline{TI} = \overline{GN} \quad m \angle TI = m \angle GN = 34 \text{ units} \]

b. Write a congruence statement for the two triangles. Is there more than one correct congruence statement? Explain.

\[ a^2 + b^2 = c^2 \]
\[ (16)^2 + (30)^2 = (c)^2 \]
\[ 256 + 900 = c^2 \]
\[ c = \sqrt{1156} = 34 \text{ units} \]
2-1. Recall that if two polygons have the same angle measures and side lengths for all pairs of corresponding angles and sides, then the two polygons are **congruent** and there is a sequence of rigid transformations that maps one polygon onto the other.

a. Verify that all the corresponding side lengths and all the corresponding angle measures of the two triangles below are equal. What is a sequence of rigid transformations that maps the triangle on the left onto its image on the right? Note: The triangles are drawn to scale.

b. Write a congruence statement for the two triangles. Is there more than one correct congruence statement? Explain.

\[ \Delta TRI \cong \Delta GAN \]
\[ \Delta RIT \cong \Delta ANG \]
2.2. Luis wants to write a statement that the two triangles below are congruent. He started with \( \triangle MNP \cong \ldots \), but got stuck because the triangles are not oriented the same way.

a. Complete Luis's statement for him and explain your reasoning.

\[ \triangle MNP \cong \triangle U VW \]

b. These triangles are drawn to scale. What sequence of rigid transformations would map the triangle on the left onto the one on the right?

Work 2-2 to 2-3, 2-2Huddle (a)
2.3. The triangles below are drawn to scale and congruent sides and angles are indicated with tick marks. Are the triangles congruent? If so, write a congruence statement and explain how you know they are congruent. Justify your answer in several ways.
2-4. In a previous course you investigated congruent triangles. Read the Math Notes box in this lesson to review the triangle congruence theorems. Then determine if the following pairs of triangles must be congruent. If they are congruent, write a congruence statement and the theorem you used, and describe a sequence of rigid transformations that would map one triangle onto the other. Note: The diagrams are not necessarily drawn to scale.

Triangles are congruent by ASA Congruency.

The red angles are congruent because of Alternate Interior Angle Theorem, while the blue angles are congruent by Vertical Angle Theorem.

Triangles are congruent by AAS Congruency.
Blue angles are congruent by Alternate Interior Angle Theorem. HF is congruent to itself by the reflexive property.
Triangle Congruence Theorems

If two polygons are congruent, then there is a sequence of rigid transformations (reflections, rotations, translations) that maps one polygon onto the other. The converse is also true: if there is a sequence of rigid transformations that maps one polygon onto the other, then the two polygons are congruent.

For triangles, you can also use specific combinations of congruent, corresponding parts as shortcuts to determine if they are congruent. These combinations, called triangle congruence theorems, are:

- **SSS**: If all three pairs of corresponding sides are congruent, then the triangles are congruent.

- **SAS**: If two pairs of corresponding sides are congruent and the angles between them (the included angle) are congruent, then the triangles are congruent.

- **ASA**: If two pairs of corresponding angles are congruent and the corresponding sides between them are congruent, then the triangles are congruent.

- **AAS**: If two pairs of corresponding angles and a pair of corresponding sides that is not between them are congruent, then the triangles are congruent.

- **HL**: If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and a leg of another right triangle, then the triangles are congruent.
\[ \sqrt{17} = z \]

\[ 4^2 + x^2 = (\sqrt{20})^2 \]

\[ 16 + x^2 = 20 \]

\[ x^2 = 4 \]

\[ x = 2 \]

\[ \sqrt{17} = z \]

\[ P = 5 + 8 + \sqrt{20} + \sqrt{17} \]

\[ P = (13 + \sqrt{17} + \sqrt{20}) \text{ m} \]