Bridge – Products, Factors, and Factor Pairs Name:_________________________

In mathematics, **factors** are numbers that create new numbers when they are multiplied. A number resulting from multiplication is called a **product**. In other words, since $2(3) = 6$, 2 and 3 are **factors** of 6, while 6 is the **product** of 2 and 3. Also, $1(6) = 6$, so 1 and 6 are two more factors of 6. Thus, the number 6 has four factors: 1,2,3, and 6. In this lesson, you will use an extended multiplication table to discover some interesting patterns of numbers and their factors.

1-73.

Have you ever noticed how many patterns exist in a simple multiplication table? Such a table is a great tool for exploring products and their factors. Fill in the missing products, without the use of a calculator.

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**
1	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30
3	3	6	9	12	15	18	21	24	27	30	33	36	39	42	45
4	4	8	12	16	20	24	28	32	36	40	44	48	52	56	60
5	5	10	15	20	25	30	35	40	45	50	55	60	65	70	75
6	6	12	18	24	30	36	42	48	54	60	66	72	78	84	90
7	7	14	21	28	35	42	49	56	63	70	77	84	91	98	105
8	8	16	24	32	40	48	54	64	72	80	88	96	104	112	120
9	9	18	27	36	45	54	63	72	81	90	99	108	117	126	135
10	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150
11	11	22	33	44	55	66	77	88	99	110	121	132	143	154	165
12	12	24	36	48	60	72	84	96	108	120	132	144	156	168	180
13	13	26	39	52	65	78	91	104	117	130	143	156	169	182	195
14	14	28	42	56	70	84	98	112	126	140	154	168	182	196	210
15	15	30	45	60	75	90	105	120	135	150	165	180	195	210	225

1-74.

Gloria was looking at the multiplication table and noticed an interesting pattern.

"Look," she said to her team. *"All of the prime numbers show up only two times as products in the table, and they are always on the edges."*

Discuss Gloria's observation with your team. Then choose one color to mark all of the prime numbers. Why does the placement of the prime numbers make sense?

Can only be a product of one and itself

1-76.

Consider the number 36 which could have been Ann's number in part (b) of problem 1-62.

a. Choose a color or design (such as circling or drawing an X) and mark every 36 that appears in the table.

b. Imagine that more rows and columns are added to the multiplication table until it is as big as your classroom floor. Would 36 appear more times in this larger table? If so, how many more times and where? If not, how can you be sure?

c. List all of the **factor pairs** of 36. (A **factor pair** is a pair of numbers that multiply to give a particular product. For example, 2 and 10 make up a factor pair of 20, because $2 \cdot 10 = 20$.) How do the factor pairs of 36 relate to where it is found in the table? What does each factor pair tell you about the possible rectangular arrays for 36? How many factors does 36 have?

b) 1x36
36x1
2x18
18x2
4 more times

Factor Pairs 36: (Arrays)
1x36
2x18
3x12
4x9
6x6

Factors of 36:
1, 2, 3, 4, 6, 9, 12, 18, 36

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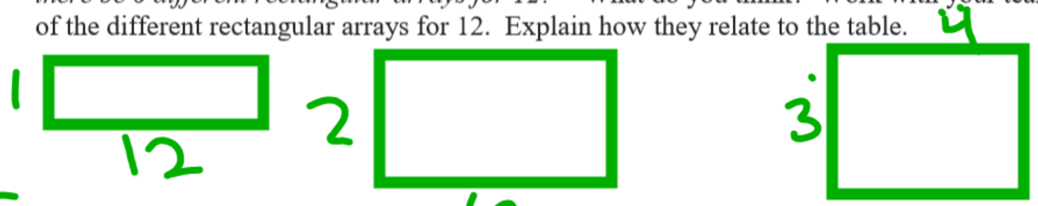
1-77.

Frequency is the number of times an item appears in a set of data. What does the frequency of a number in the multiplication table tell you about the rectangular arrays that are possible for that number?

a. Gloria noticed that the number 12 appears, as a product, 6 times in the table. She wonders, *"Shouldn't there be 6 different rectangular arrays for 12?"* What do you think? Work with your team to draw all of the different rectangular arrays for 12. Explain how they relate to the table.



b. How many different rectangular arrays can be drawn to represent the number 48? How many times would 48 appear as a product in a table as big as the classroom? Is there a relationship between these answers?

12 x 4 24 x 2 16 x 3 5 arrays, appears 10 times

8 x 6 48 x 1

c. In problem 1-76 how many different rectangular arrays could be drawn to represent the number 36? How many times did it appear as a product in a table as big as the room?

Does the pattern you noticed for 12 and 48 apply to 36? If so, why does this make sense? If not, why is 36 different?

1-78.
PRIME FACTORIZATION

8x25

200x1 Factors of 200: 1, 2, 4, 5, 8, 10, 20, 25, 40, 50, 100, 200

a. What are all the factors of 200?

4x50 10x20 5x40 100x2

b. A **prime factor** is a factor that is also a prime number. What are the prime factors of 200?

Prime factors of 200: 2, 5

c. Writing a number as a product of only prime numbers is called **prime factorization**. Tatiana was writing 200 as a product of prime numbers. She shared with her team the beginning of her work, which is shown at right.

$$200$$
$$200 = 4 \cdot 50$$
$$200 = 2 \cdot 2 \cdot 10 \cdot 5$$
$$200 = 2 \cdot 2 \cdot 5 \cdot 2 \cdot 5$$

Notice that Tatiana uses a "dot" (·) to represent multiplication. This is a way to show multiplication without using an "x". Try to use this method now so that when you learn algebra, you are not confused about the use of the letter x as a variable. What process do you think was going through Tatiana's mind when she wrote 200 as a product of prime factors?

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d. Do you think it matters what products Tatiana wrote in her second step? What if she wrote 10·20 instead? Finish this **prime factorization** using Tatiana's process.

200

$= 10 \cdot 20$

$= 5 \cdot 2 \cdot 5 \cdot 4$

$\underbrace{= 5 \cdot 2 \cdot 5 \cdot 2 \cdot 2}_{\text{Prime Factorization}}$

200

$= 10 \cdot 20$

$= 5 \cdot 2 \cdot 2 \cdot 10$

$= 5 \cdot 2 \cdot 2 \cdot 5 \cdot 2$

1-79.
Write the prime factorization of each of the numbers below.

a. 100

$= 50 \cdot 2$

$= 25 \cdot 2 \cdot 2$

$= 5 \cdot 5 \cdot 2 \cdot 2$

b. 36

$= 6 \cdot 6$

$= 2 \cdot 3 \cdot 2 \cdot 3$

c. 54

d. 600

$\boxed{\text{1-80 (b, c)}}$

b. To make it easier to record prime factors, you can use exponents.

Do you remember how repeated addition can be written in shorter form using multiplication? For example, 10+10+10+10+10 can be written as $5 \cdot 10$. Similarly, repeated multiplication can be written in shorter form using exponents: $10 \cdot 10 \cdot 10 \cdot 10 \cdot 10 = \cancel{1X} \cdot 10^5$

The prime factorization of 200 from problem 1-78 was $\underbrace{2 \cdot 2 \cdot 2}\cdot\underbrace{5 \cdot 5}$. How could you write this with exponents?

$200 = 2^3 \cdot 5^2 \cdot$

c. Write your answers from problem 1-79 in exponent form.

a. 100

$= 50 \cdot 2$

$= 25 \cdot 2 \cdot 2$

$= 5 \cdot 5 \cdot 2 \cdot 2$

$= 5^2 \cdot 2^2$

b. 36

$= 6 \cdot 6$

$= 2 \cdot 3 \cdot 2 \cdot 3$

$= 2^2 \cdot 3^2$